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Annex D: Reliability of geotechnical structures

D.1 Introduction

The purpose of Annex D is to identify and characterize critical elements of the geotechnical reliability-based design process that differ from the main body of this standard. These elements cannot be accounted for in existing deterministic geotechnical practice as well. The critical distinctive elements are:

(1) Coefficients of variation (COVs) of geotechnical parameters can be potentially large, because geomaterials are naturally occurring and in-situ variability cannot be reduced (in contrast, most structural materials are manufactured with quality control).

(2) COVs for geotechnical design parameters are not unique and can vary over a wide range, depending on the procedure in which they are derived.

(3) It is common to conduct both laboratory and field tests in a site investigation. A geotechnical design parameter is typically correlated with more than one laboratory and/or field test indices. It is important to consider this multivariate correlation structure where possible, because the COV of the design parameter reduces when consistent information increases.

(4) Spatial variability of geotechnical parameters cannot be readily dismissed, because the volume of geomaterial interacting with the structure is related to some multiple of the characteristic length of the structure and this characteristic length (e.g., height of slope, diameter of tunnel, depth of excavation) is typically larger than the scale of fluctuation of the design parameter, particularly in the vertical direction.

(5) There are usually many different geotechnical calculation models for the same design problem. Hence model calibration based on local field tests and local experience is important. The proliferation of model factors, possibly site-specific, is to be expected, because of the number of models and the number of calibration databases.

(6) A geotechnical system, such as a pile group and a slope, is a system reliability problem containing multiple correlated failure modes. Some of these problems are further complicated by the fact that the failure surfaces are coupled to the spatial variability of the soil medium.

Although the general principles of reliability-based design are applicable to geotechnical engineering, it is essential to cater for a wide range of COVs and other site conditions in geotechnical RBD to avoid over-simplification of ground truths and to be in line with prevailing geotechnical engineering practice that respects a diversity of methodologies.

D.2 Coefficients of variation of design soil parameters

D.2.1 Estimation of design soil parameters

The COV of a design soil parameter is not an intrinsic statistical property. It depends on the site condition, the measurement method, and the transformation (correlation) model. Hence, the COV takes a range of values rather than a unique value. Some guidelines on the typical COVs of common design soil parameters as a *function* of soil type, measurement method, and transformation model are below.

D.2.2 Sources of uncertainties

The overall uncertainty underlying a geotechnical design parameter results from many disparate sources of uncertainties, as illustrated in Figure D-1. There are three primary sources of geotechnical uncertainties: (1) inherent variability, (2) measurement error, and (3) transformation uncertainty. The first results primarily from the natural geologic processes that produced and continually modify the soil mass in-situ. The second is caused by equipment, procedural/operator, and random testing effects. The third source of uncertainty is introduced when field or laboratory measurements are transformed into design soil parameters using empirical or other correlation models.

D.2.3 Inherent variability

Typical inherent variability of strength properties, index parameters, and field measurements are given in Tables 1-3 in Phoon & Kulhawy (1999a,b). Site-specific inherent variability statistics are reported by Cherubini et al. (2007), Chiasson & Wang (2007), Jaksa (2007), and Uzielli et al. (2007).

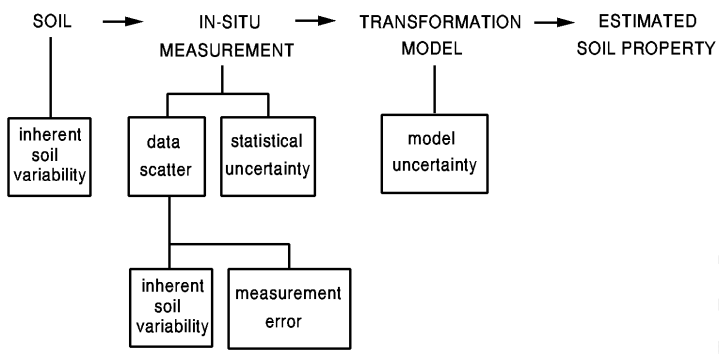

Figure D-1. Sources of uncertainties contributing to overall uncertainty in design soil parameter (Phoon & Kulhawy, 1999a)

D.2.4 Measurement error

Typical measurement error of laboratory tests and field tests are given in Tables 5-6 in Phoon & Kulhawy (1999a,b).

D.2.5 Transformation uncertainty

Transformation models are needed to relate test measurements to appropriate design properties. Uncertainty is introduced, because most transformation models in geotechnical engineering are obtained by empirical or semi-empirical data fitting. The transformation model is typically evaluated using regression analyses and the spread of the data about the regression curve is modelled as a zero-mean random variable (ε). The standard deviation of ε is an indicator of the magnitude of transformation uncertainty.

Most transformation models were developed for a specific geomaterial type and/or a specific locale. Site-specific models are generally more precise than “global” models calibrated from data covering many sites (Ching & Phoon 2012a). However, site-specific models can be significantly biased when applied to another site. This “site-specific” limitation is a distinctive and prevalent feature of geotechnical engineering practice. Geotechnical RBD must take cognizance of this limitation to avoid gross oversimplification of “ground truths”.

D.2.6 Total uncertainty

Uncertainty in a design soil parameter is a function of inherent soil variability, measurement error, and transformation uncertainty. These components can be combined consistently using a simple second-moment probabilistic approach described in Phoon & Kulhawy (1999b) or by more sophisticated uncertainty analysis methods. First-order approximate guidelines for COVs of some design soil parameters as a function of the test measurement, correlation equation, and soil type are presented in Table D-1. An illustrative COV for a spatial average over 5 m is also presented in the table to highlight the critical need to identify the characteristic design parameter governing a specific limit state. For ultimate limit state problems, this characteristic design parameter is typically a spatially averaged strength over the most critical failure path. In the presence spatial variability, the COV of this spatially averaged strength is smaller than the COV of the point strength; the degree of COV reduction is a function of the scale of fluctuation discussed in Section D.2.7.

It is inappropriate to apply the point COV to a problem where the COV reduction is significant, say because the scale of fluctuation is short relative to some characteristic length scale of the failure path length (e.g., height of slope, diameter of tunnel, depth of excavation).

Table D-1. Approximate guidelines for coefficients of variation of some design soil parameters (Source: Table 5, Phoon & Kulhawy 1999b)

Design Property[a]	Test[b]	Soil type	Point COV (%)	Spatial Avg. COV[c] (%)	Correlation Equation[f]
su(UC)	Direct (lab)	Clay	20-55	10-40	-
su(UU)	Direct (lab)	Clay	10-35	7-25	-
su(CIUC)	Direct (lab)	Clay	20-45	10-30	-
su(field)	VST	Clay	15-50	15-50	14
su(UU)	qT	Clay	30-40d	30-35d	18
su(CIUC)	qT	Clay	35-50d	35-40d	18
su(UU)	N	Clay	40-60	40-55	23
sue	KD	Clay	30-55	30-55	29
su(field)	PI	Clay	30-55d	-	32
$\bar{\phi}$	Direct (lab)	Clay, sand	7-20	6-20	-
$\bar{\phi}$ (TC)	qT	Sand	10-15d	10d	38
$\bar{\phi}_{cv}$	PI	Clay	15-20d	15-20d	43
Ko	Direct (SBPMT)	Clay	20-45	15-45	-
Ko	Direct (SBPMT)	Sand	25-55	20-55	-
Ko	KD	Clay	35-50d	35-50d	49
Ko	N	Clay	40-75d	-	54
EPMT	Direct (PMT)	Sand	20-70	15-70	-
ED	Direct (DMT)	Sand	15-70	10-70	-
EPMT	N	Clay	85-95	85-95	61
ED	N	Silt	40-60	35-55	64

a - su = undrained shear strength; UU = unconsolidated-undrained triaxial compression test; UC = unconfined compression test; CIUC = consolidated isotropic undrained triaxial compression test; su(field) = corrected su from vane shear test; $\bar{\phi}$ = effective stress friction angle; TC = triaxial com-pression; $\bar{\phi}_{CV}$ = constant volume $\bar{\phi}$; Ko = in-situ horizontal stress coefficient; EPMT = pressure-meter modulus; ED = dilatometer modulus
b - VST = vane shear test; qT = corrected cone tip resistance; N = standard penetration test blow count; KD = dilatometer horizontal stress index; PI = plasticity index
c - averaging over 5 m
d - COV is a function of the mean; refer to COV equations in text for details
e - mixture of su from UU, UC, and VST
f - Equation numbering in Phoon & Kulhawy (1999b)

It suffices to note that it is not realistic to assign a single representative value to a design soil parameter. For example, a COV of 30% for undrained shear strength may be appropriate for good quality laboratory measurements or direct correlations from field measurements such as the cone penetration test. It may not be appropriate for indirect correlations based on the standard penetration test. The practical impact of this observation is that it is unlikely to be realistic to calibrate a single resistance or partial factor for application in the context of a simplified RBD format. Based on reliability calibration studies for foundations (Phoon et al. 1995), Kulhawy et al. (2012) proposed a reasonably practical three-tier classification scheme to approximately account for the range of COVs encountered in geotechnical design parameters for development of simplified RBD formats, such as the Load Resistance Factor Design (LRFD) format, in geotechnical engineering (Table D-2).

Table D-2. Three-tier classification scheme of soil property variability for reliability calibration (Source: Table 3, Kulhawy et al. 2012)

Geotechnical parameter	Property variability	COV (%)
Undrained shear strength	Low[a]	10 - 30
	Medium[b]	30 - 50
	High[c]	50 - 70
Effective stress friction angle	Low[a]	5 - 10
	Medium[b]	10 - 15
	High[c]	15 - 20
Horizontal stress coefficient	Low[a]	30 - 50
	Medium[b]	50 - 70
	High[c]	70 - 90

a - typical of good quality direct lab or field measurements
b - typical of indirect correlations with good field data, except for the standard penetration test (SPT)
c - typical of indirect correlations with SPT field data and with strictly empirical correlations

D.2.7 Scale of fluctuation

As illustrated in Figure D-2, this spatial variation can be decomposed conveniently into a smoothly varying trend function [t(z)] and a fluctuating component [w(z)]. This fluctuating component is typically modelled as a statistically homogeneous random field (Vanmarcke 1977). It is noteworthy that a physically homogeneous soil layer is not necessarily statistically homogeneous. A critical statistical parameter that is needed to describe inherent variability is the correlation distance or scale of fluctuation (Vanmarcke 2010). The scale of fluctuation provides an indication of the distance within which the property values show relatively strong correlation. A simple but approximate method of determining the scale of fluctuation is shown as an insert in Figure D-2.

A summary of scales of fluctuation is reported in Table 4 in Phoon & Kulhawy 1999a. Detailed studies on the scale of fluctuation are available, but rather limited in number (Jaksa 1995, Fenton 1999, Uzielli et al. 2005).

The practical importance of considering a reasonable scale of fluctuation, i.e. a reasonably realistic spatial variability, in the estimation of COV has been highlighted in Section D.2.6. The assumption of independent soil parameters will produce an unconservative reduction of the point COV for the spatial average. The assumption of fully correlated soil parameters will not result in COV reduction for the spatial average, which is overly conservative. In addition to COV reductions, it is important to point out that failure mechanisms are related to spatial variability. Examples are provided by Fenton & Griffiths (2008).

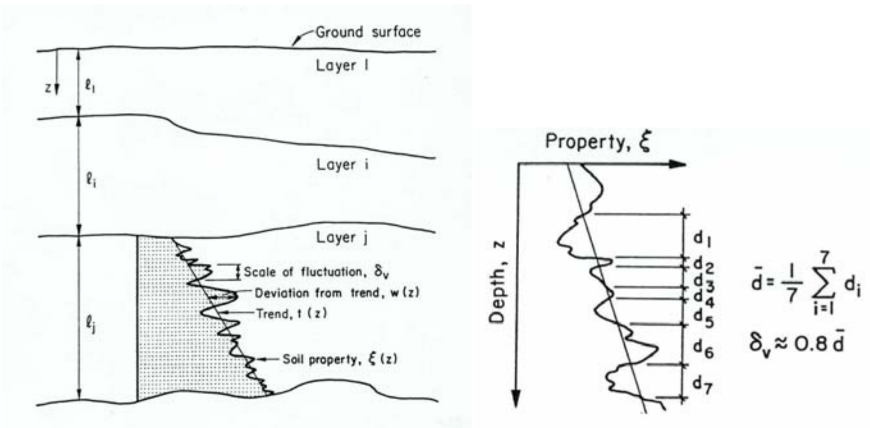

Figure D-2. Random field model for inherent soil variability (revised from Phoon & Kulhawy 1999a)

D.3 Statistical characterization of multivariate geotechnical data

Section D.2 focuses on COVs of design soil parameters. In particular, Table D-1 in Section D.2.4 is useful to determine COVs of design soil parameters based on univariate information from a laboratory or in-situ tests. Nonetheless, multivariate information is usually available in a typical site investigation. For instance, when undisturbed samples are extracted for oedometer and triaxial tests, SPT and/or piezocone test (CPTU) may be conducted in close proximity. Moreover, data sources such as the unit weight, plastic limit (PL), liquid limit (LL), and liquidity index (LI) are commonly determined from relatively simple laboratory tests on disturbed samples. A number of these data sources could be simultaneously correlated to a measure of the undrained shear strength (s_u).

Because bivariate correlations between soil parameters are more commonly available, for example s_u-N and s_u-OCR, the multivariate normal distribution is a sensible and practical choice to capture the multivariate dependency among soil parameters in the presence of transformation uncertainties (Phoon et al. 2012). A collection of bivariate correlations $\{\delta_{ij}$: i = 1,…n-1, j = i+1,…n$\}$ for five parameters relevant to structured clays is shown in Table D-3. The practical significance of considering multivariate data is that the COV of a design parameter typically decreases when it is estimated from more than one parameter. Hence, site investigation is not a cost item but an investment item, because reduction of uncertainties through multivariate tests can translate directly to design savings through RBD. This important link between the quality/quantity of site investigation and design savings cannot be addressed systematically in a deterministic design approach.

Table D-3. Correlation matrix C for (LI, s_u, s_u^{re}, σ'_p, σ'_v) estimated from a structured clay database containing 345 data points from 37 sites worldwide (Source: Table 4, Ching & Phoon 2012b).

		LI	su	sure	for σ'p	σ'v
	LI	1.000	δ12 = -0.083	δ13 = -0.824	δ14 = -0.176	δ15 = -0.280
	su		1.000	δ23 = 0.276	δ24 = 0.915	δ25 = 0.801
C =	sure			1.000	δ34 = 0.365	δ35 = 0.453
	σ'p		Symmetric		1.000	δ45 = 0.850
	σ'v					1.000

Notes: LI = liquidity index, s_u = undrained shear strength, s_u^{re} = remolded s_u, σ'_p = preconsolidation stress, and σ'_v = effective vertical stress.

D.4 Statistical characterization of model factors for foundations

D.4.1 Definition of model factor

This section presents model statistics for common foundation engineering problems: (1) laterally loaded rigid bored piles (ultimate limit state), (2) axially loaded piles (ultimate limit state), (3) shallow foundations (ultimate limit state), (4) axially loaded piles (serviceability limit state), and (5) limiting tolerable displacement (serviceability limit state).

The model factor for the capacity of a foundation is commonly defined as the ratio of the measured (or interpreted) capacity to the calculated capacity:

$$M = \frac{Q_i}{Q_p} \qquad \text{(D-1)}$$

in which Q_i = "capacity" interpreted from a load test (e.g., hyperbolic capacity), Q_p = capacity generally predicted using limit equilibrium models, and M = model factor, assumed to be a log-normal random variable.

D.4.2 Laterally loaded rigid bored piles (ultimate limit state)

The capacity of a laterally loaded pile is typically predicted using conventional ultimate lateral soil stress models. The statistics of the model factor for different ultimate lateral soil stress models under undrained and drained loading modes are reported in Table 2 in Phoon & Kulhawy (2005).

D.4.3 Axially loaded piles (ultimate limit state)

Tables D-4 and D-5 present the model factors statistics for the capacity of axially loaded piles for various calculation methods, soil conditions, and failure interpretation methods. The model factors are defined following Eq. (D-1).

Table D-4: Model factors for driven piles

Calculation method	No. of cases	Pile type	Soil type	Mean	COV	Source
β-method	4	H-pile		0.61	0.61	
λ-method	16			0.74	0.39	
α-Tomlinson	17		Clay	0.82	0.40	
α-API	16			0.90	0.41	
SPT-97 mob	8			1.04	0.39	
λ-method	18	Concrete piles		0.76	0.39	
α-API	17			0.81	0.36	
β-method	8		Clay	0.81	0.31	
α-Tomlinson	18			0.87	0.48	
α-Tomlinson	18	Pipe piles		0.64	0.50	
α-API	19			0.79	0.54	
β-method	12		Clay	0.45	0.60	
λ-method	19			0.67	0.55	
SPT-97 mob	12			0.39	0.62	
Nordlund	19	H-pile		0.94	0.4	NCHRP Report 507
Meyerhof	18		Sand	0.81	0.38	
β-method	19			0.78	0.51	
SPT-97 mob	18			1.35	0.43	
Nordlund	36	Concrete piles	Sand	1.02	0.48	

β-method	35			1.1	0.44	
Meyerhof	36			0.61	0.61	
SPT-97 mob	36			1.21	0.47	
Nordlund	19	Pipe piles	Sand	1.48	0.52	
β-method	20			1.18	0.62	
Meyerhof	20			0.94	0.59	
SPT-97 mob	19			1.58	0.52	
α-Tomlinson/Nordlund/Thurman	20	H-pile	Mixed soils	0.59	0.39	
α-API/Nordlund/Thurman	34			0.79	0.44	
β-method/Thurman	32			0.48	0.48	
SPT-97 mob	40			1.23	0.45	
α-Tomlinson/Nordlund/Thurman	33	Concrete piles	Mixed soils	0.96	0.49	
α-API/Nordlund/Thurman	80			0.87	0.48	
β-method/Thurman	80			0.81	0.38	
SPT-97 mob	71			1.81	0.50	
FHWA CPT	30			0.84	0.31	
α-Tomlinson/Nordlund/Thurman	13	Pipe piles	Mixed soils	0.74	0.59	
α-API/Nordlund/Thurman	32			0.8	0.45	
β-method/Thurman	29			0.54	0.48	
SPT-97 mob	33			0.76	0.38	
Static formula	28	Concrete piles	Sand	1.11	0.33	Dithinde et al 2011
Static formula	59	& H piles	Clay	1.17	0.26	
Meyerhof	24		Sand	1.22	0.54	FHWA-HI-98-032

Table D-5: Model factors for bored piles

Calculation method	Constr. Method[a]	No. of cases	Soil type	Mean	COV	Source
Static formula	Mixed	30	Sand	0.98	0.24	Dithinde et al 2011
Static formula		53	Clay	1.15	0.25	
FHWA (1999)	Casing	11	Sand/silt	0.60	0.58	Zhang & Chu 2009a
FHWA (Hong Kong data)	Casing	17	Sand/silt	1.06	0.28	Zhang & Chu 2009a
FHWA (1999)	RCD	15	Rocks	0.48	0.52	Zhang & Chu 2009a
COP (BD 2004)	RCD	15	Rocks	2.57	0.31	Zhang & Chu 2009a
FHWA (1999)	Mixed	32	Sand	1.71	0.60	NCHRP Report 507
	Casing	12		2.27	0.46	
	Slury	9		1.62	0.74	
R&W	Mixed	32		1.22	0.67	
	Casing	12	Sand	1.45	0.5	
	Slury	9		1.32	0.62	
FHWA (1999)	Mixed	53	Clay	0.9	0.47	
	Casing	14		0.84	0.50	
	Dry	30		0.88	0.48	

FHWA (1999)	Mixed	44	Clay + Sand	1.19	0.30
	Casing	21		1.04	0.29
	Dry	12		1.32	0.28
	Slury	10		1.29	0.27
R&W	Mixed	44		1.09	0.35
	Casing	21	Clay + Sand	1.01	0.42
	Slury	12		1.2	0.32
	Slury	10		1.16	0.25
C&K	Mixed	46	Rock	1.23	0.40
	Dry	29		1.29	0.34
IGM	Mixed	46	Rock	1.3	0.34
	Dry	29		1.35	0.31

a - Casing = pile bore excavation assisted by steel casing; RCD = reverse circulation drilling in rocks; slurry = excavation assisted by mineral slurry; dry = excavation above groundwater.

D.4.4 Shallow foundations (ultimate limit state)

Model factor statistics for shallow foundations are scarce compared to pile foundations. Model statistics for different loading modes (vertical eccentric loading, inclined eccentric loading) are reported in NCHRP 651 (2010).

D.4.5 Axially loaded foundations (serviceability limit state)

Limit state design requires that the occurrence of both ultimate and serviceability limit states are sufficiently improbable. For consistency, it is imperative that serviceability limit state verification be based on reliability principles.

Tables D-6 and D-7 present values of model factor for driven steel H-piles and bored piles, respectively. The model factor here refers to the ratio of the measured pile settlement at the working load level and the predicted settlement at the same load level. Model factor statistics for allowable settlement = 25 mm from another SLS study are presented in Table D-8. In this study, the SLS model statistics were derived by fitting measured load-settlement data to a hyperbolic equation. This approach is practical and grounded realistically on the load test database with minimal assumptions. The mean values in Tables D-6 to D-8 are different, because the calculation methods are different. This is to be expected. It is more interesting to observe that the COVs are comparable, despite the diverse variety of calculation methods.

Table D-6: Model factors for driven piles at the working load level defined as one half of Davisson's capacity (Source: Zhang et al. 2008)

Calculation method	No. of cases	Soil type	Mean	COV
Vesic (1977)	34	Sand/silt	1.02	0.23
Fleming et al. (1992)	34	Sand/silt	0.66	0.22
Load transfer method	34	Sand/silt	1.34	0.22
Vesic (1977)	30	Rocks	0.96	0.27
Fleming et al. (1992)	30	Rocks	0.81	0.28
Load transfer method	30	Rocks	1.16	0.24

Table D-7: Model factors for large-diameter bored piles at the working load level defined as one half of Davisson's capacity (Source: Zhang & Chu 2009b)

Calculation method	Construction Method	No. of cases	Soil type	Mean	COV

Vesic (1977)	Casing	20	Sand/silt	0.24	0.38
Mayne and Harris (1993)	Casing	12	Sand/silt	0.64	0.22
Reese and O'Neill (1989)	Casing	19	Sand/silt	1.80	0.31
Vesic (1977)	RCD	14	Rocks	0.87	0.30
Kulhawy and Carter (1992)	RCD	14	Rocks	1.01	0.24
Load transfer method using correlation with RQD	RCD	14	Rocks	1.21	0.30

Table D-8: SLS model factor statistics for allowable settlement = 25 mm (After Dithinde et al 2011)

Case	N	Actual statistics				Statistics from first-order second-moment approximations			
		Ms		MsM		Ms		MsM	
		μ	COV	μ	COV	μ	COV	μ	COV
D-NC	28	1.084	0.077	1.202	0.351	1.076	0.082	1.195	0.340
B-NC	30	1.079	0.083	1.057	0.238	1.094	0.114	1.072	0.266
D-C	59	1.082	0.047	1.259	0.260	1.083	0.049	1.267	0.265
B-C	53	1.077	0.063	1.236	0.250	1.073	0.059	1.234	0.257

D-NC =driven piles in non-cohesive soils, B-NC = bored piles in non-cohesive soils, D-C = driven piles in cohesive soils, B-C = bored piles in cohesive soils, Ms = model factor for SLS, MsM=combined statistics

At the ultimate limit state, a consistent load test interpretation procedure should be used to produce a single "capacity" from each measured load-displacement curve". The ratio of the measured (or interpreted) capacity to the computed capacity is called a model factor as defined in Eq. (D-1). The same approach applies to the serviceability limit state (SLS). The capacity is replaced by an allowable capacity that depends on the allowable displacement. The distribution of the SLS model factor is established from a load test database in the same way. Notice that the SLS model factor has to be re-evaluated when a different allowable displacement is prescribed. Hence, it is important to note that Tables D-6 to D-8 only apply to an allowable settlement = 25 mm.

If the allowable settlement is treated as a random variable in the serviceability limit state, a more general approach involving fitting measured load-displacement data to a normalized hyperbolic curve is recommended as detailed below:

$$\frac{F}{Q_i} = \frac{y}{a + by} \qquad \text{(D-2)}$$

in which F = applied load, Qi= failure load or capacity interpreted from a measured load-displacement curve, "a" and "b" = curve-fitting parameters, and y = pile butt displacement. The curve-fitting equation is empirical and other functional forms can be considered (Phoon & Kulhawy, 2008). However, the important criterion is to apply a curve-fitting equation that produces the least scatter in the measured normalized load-displacement curves. Each measured load-displacement curve is thus reduced to two curve-fitting parameters. Based on "a" and "b" statistics estimated from load test databases (Table D-9), one can construct an appropriate bivariate probability distribution for (a, b) that can reproduce the scatter in the normalized load over the full range of displacements (Phoon & Kulhawy, 2008). Akbas & Kulhawy (2009) presented a probabilistic model for differential settlement of shallow foundations.

Table D-9: Statistics for hyperbolic parameters [Source: ACIP under axial compression (Phoon et al. 2006); Spread foundation, drilled shaft, pressure-injected footing under axial uplift (Phoon et al. 2007); Driven piles in non-cohesive soil, Bored piles in non-cohesive soils, Driven piles in cohesive soils, and Bored piles in cohesive soil (Dithinde et al, 2011)]

Augered cast-in-place pile (compression) No. tests = 40 a: Mean = 5.15 mm, S. D. = 3.07 mm, COV = 0.60 b: Mean = 0.62, S. D. = 0.16, COV = 0.26 Correlation = -0.67	Spread footing (uplift) No. tests = 85 a: Mean = 7.13 mm, S. D. = 4.66 mm, COV = 0.65 b: Mean = 0.75, S. D. = 0.14, COV = 0.18 Correlation = -0.24
Bored pile (uplift) No. tests = 48 a: Mean = 1.34 mm, S. D. = 0.73 mm, COV = 0.54 b: Mean = 0.89, S. D. = 0.063, COV = 0.07 Correlation = -0.59	Pressure injected footing (uplift) No. tests = 25 a: Mean = 1.38 mm, S. D. = 0.95 mm, COV = 0.68 b: Mean = 0.77, S. D. = 0.21, COV = 0.27 Correlation = -0.73
Driven piles in non-cohesive soils (compression) No. tests = 28 a: Mean = 5.55 mm, S. D. = 3.00 mm, COV = 0.54 b: Mean = 0.71, S. D. = 0.10, COV = 0.14 Correlation = -0.778	Bored piles in non-cohesive soils (compression) No. tests = 30 a: Mean = 4.10 mm, S. D. = 3.20 mm, COV = 0.78 b: Mean = 0.77, S. D. = 0.16, COV = 0.21 Correlation = -0.876
Driven piles in cohesive soils (compression) No. tests = 59 a: Mean = 3.58 mm, S. D. = 2.04 mm, COV = 0.57 b: Mean = 0.78, S. D. = 0.09, COV = 0.11 Correlation = -0.886	Bored piles in non-cohesive soils (compression) No. tests = 53 a: Mean = 2.79 mm, S. D. = 2.04 mm, COV = 0.57 b: Mean = 0.82, S. D. = 0.09, COV = 0.11 Correlation = -0.801

D.4.6 Limiting tolerable displacement (serviceability limit state)

Another important serviceability consideration is limiting tolerable displacements of structures. The limiting tolerable displacements of a structure are affected by many factors, including the type and size of the structure, the intended usage of the structure, substructure–superstructure interactions, the properties of the structural materials and the subsurface soils, and the rate and uniformity of settlement (Zhang & Ng 2005). For a full reliability-based design for serviceability limit states, it is preferable to obtain the probability distributions of limiting tolerable displacement. Tables D-10 and D-11 provide statistics of intolerable settlement and angular distortion, and limiting tolerable settlement and angular distortion of buildings based on records of the displacements of 380 buildings. The intolerable or limiting tolerable displacements are shown to follow the lognormal distribution.

Table D-10: Statistics of intolerable settlement and limiting tolerable settlement of buildings (Source: Table 3, Zhang & Ng 2007)

	No. of cases	Observed intolerable settlement (mm)		Limiting tolerable settlement (mm)	
		Mean	Standard deviation	Mean	Standard deviation
Foundation type					
All	221	328	265	156	118
Shallow foundations	165	321	280	218	185
Deep foundations	52	349	218	106	55
Structural type					
All	185	296	220	134	109
Frame structures	115	278	236	148	126
With load-bearing wall	52	303	257	112	48
Soil type					
All	182	311	270	165	159
Clay	126	357	290	169	131
Sand and fill	56	207	151	86	56
Usage of building					
All	164	269	247	150	144
Mill structure	29	308	193	183	156
Office structure	135	255	265	121	64

Table D-11: Statistics of intolerable and limiting tolerable angular distortion (Source: Table 4, Zhang & Ng 2007)

Statistics	No. of cases	Observed intolerable angular distortion (radian)		Limiting tolerable angular distortion (radian)	
		Mean	Standard deviation	Mean	Standard deviation
Foundation type					
All	120	0.012	0.012	0.003	0.003
Shallow foundations	63	0.013	0.011	0.006	0.006
Deep foundations	57	0.008	0.011	0.002	0.002
Structural type					
All	191	0.012	0.014	0.004	0.005
Frame structures	152	0.011	0.015	0.005	0.004
With load-bearing wall	39	0.015	0.011	0.004	0.002
Soil type					
All	126	0.011	0.013	0.006	0.014
Clay	103	0.011	0.011	0.005	0.014
Sand and fill	23	-	-	-	-
Usage of building					
All	83	0.015	0.013	0.005	0.003
Mill structure	17	0.032	0.026	0.006	0.003
Office structure	66	0.013	0.013	0.003	0.002

D.5 Implementation issues in geotechnical reliability-based design

D.5.1 Goal and challenges

Although the general principles in the main text are equally applicable to geotechnical reliability-based design (RBD), it is important to highlight that reliability calibration is more challenging in geotechnical engineering. One key reason is the diversity of design scenarios that must be considered in the calibration domain, such as the range of COV resulting from different soil property estimation methodologies. Another source of diversity is the number of different soil profiles encountered even within a city size locale. The key goal in geotechnical RBD must be to achieve a more uniform level of reliability than that is implied in existing allowable stress design.

The challenge of producing geotechnical RBD that is both realistic (handle wide range of design scenarios) and robust (achieve target reliability index with reasonable accuracy) is being addressed in this section. Some implementation details associated with semi probabilistic methods (i.e., partial factor methods and characteristic values in Clause 8) and probability based decision making (i.e., direct probability- based design, system reliability and reliability targets in Clause 7) are discussed below.

D.5.2 Characteristic value

The concept of a “characteristic value” is intrinsically link to simplified reliability-based formats, particularly the partial factor approach (cf. Clause 2.4.1). In this approach, using the ultimate limit state as an example, a “characteristic value” for a soil parameter (e.g., undrained shear strength), is divided by a strength partial factor to produce a “design” value (cf. Clause 2.4.3) and the geotechnical capacity based on this design value should be larger than the design load (characteristic load multiplied by a load factor).

It is evident that the concept of characteristic value is not necessary if reliability-based design (RBD) is undertaken by carrying out reliability analyses, rather than applying simplified design formats. The required inputs for a reliability analysis are the probability distributions of the soil parameters and the loads. The identification of appropriate probability distributions and the selection of appropriate model parameters for these distributions (e.g., mean and COV) from limited data is subject to statistical uncertainties.

Regardless of whether RBD is carried out using simplified formats or more rigorous reliability analyses, the soil parameter must be defined such that it is relevant to the limit state equation. For example, if a single undrained shear strength parameter appears in a slope stability equation, then the relevant physical definition is the spatial average along the most critical failure path. It is neither the undrained shear strength at a point in the soil mass nor a spatial average along a prescribed line in the soil mass. The latter has been studied fairly extensively (Vanmarcke 2010) while the former is gradually being recognized (Ching & Phoon 2012c).

The emphasis in the geotechnical literature is on clarifying this physical aspect of the characteristic value, which is justifiably so.

It is necessary to make clear the physical meaning of the characteristic soil parameter before the uncertainty aspect could be rationally considered. For illustration, the characteristic undrained shear strength for the shaft friction of a pile is the spatial average along the length of the pile, while the characteristic undrained shear strength for the end bearing of a pile is the spatial average within a bulb of soil below the pile tip. When reliability analysis is carried out, the performance function will contain two random variables following two distinct probability distributions for these spatial averages.

When simplified RBD is carried out, it would be necessary to select a single value characteristic of each probability distribution. This value may refer to the mean or to the lower 5% quantile. The statistical estimation of these characteristic values is subject to the same statistical uncertainties underlying the probability distributions appearing in reliability analysis. Clearly, this statistical aspect of the characteristic value is distinctive from the physical aspect of the characteristic value. The emphasis in the structural literature is on this statistical aspect of the characteristic value (cf. Section 2.4.1), in part because the physical aspect is rather unambiguous in structural design problems.

In principle, partial factors can be calibrated to achieve a prescribed target reliability index for any statistical definition of the characteristic value. In practice, it is known that a partial factor calibrated for the mean value could change significantly if the COV of the input random variable changes. This limitation is less severe for a partial factor calibrated using say the lower 5%-quantile. Hence, if the COV of an input random variable varies over a wide range within the scope of design scenarios covered in a design code and if there is a practical need to simplify presentation of a partial factor as a single number rather than as a function of COV, the lower 5% quantile definition is preferred. It is useful to reiterate that the key function of a RBD code is to achieve a prescribed target reliability index (typically a function of limit state and importance of structure) over a range of commonly encountered design scenarios and not for a specific design scenario. The statistical definition of the characteristic value should be viewed within this broader context of what a RBD code intends to achieve, rather than adherence to past practice or a component separate from reliability calibration. In other words, the ensuing design is produced by design values, which is the product of partial/resistance factors with characteristic values, not merely characteristic values alone. There are practical concerns regarding: (1) estimation of quantiles reliably from limited data and (2) quantiles falling below known lower bounds (e.g., residual friction angle) due to inappropriate choice of unbounded probabilitiy distribution functions. However, both concerns do not merely affect the characteristic value but fundamentally affect the reliability analyses underling code calibration as well.

D.5.3 Partial factor method

In the calibration process, the "best" geotechnical (deterministic) calculation models should be used directly, rather than artificially simplifying the capacity into a single lognormal random variable to fit the requirement of a closed-form reliability formula (Kulhawy et al. 2012). As discussed in Sections D.1 and D.4, one distinctive feature in geotechnical design is that many geotechnical calculation models are relatively “simple” and there are usually many different calculation models for the same design problem because of the empiricism nature of our geotechnical heritage. For example, there are at least more than ten different calculation models to estimate bearing capacity of a simple footing. Some are based on field test results (e.g., SPT-N values, CPT results) directly; some are calculated from soil properties; and others may be prescribed values based on local experiences.

It is desirable to choose a “best” calculation model (minimum bias and least variability when compared against field measurements or perhaps laboratory tests/numerical simulations in the absence of field measurements) for reliability calibrations. Nonetheless, local practice must be respected in geotechnical engineering because the behavior of geomaterials and construction/design experience are largely site specific. Hence, it is more realistic to define a “best” model as what is best practice within a locale. The upshot is that it is not possible to enforce a single physical model (e.g., CPT method) or a single probabilistic model (e.g., lumped entire capacity into a single lognormal variable) and a geotechnical RBD calibration process must be sufficiently general to accommodate the existing diversity of local practices. For RBD calibration, the only mandatory requirement is that the model bias should be quantified in some approximate way.

Because of the considerably larger COV for geotechnical design parameters (see Section D.2), it is sensible to divide the range of an influential design parameter, and its variability, into several segments or domains (see Figure D-3). Then calibration points within each segment or domain can be selected to ensure uniform coverage of the variables during the calibration process and to achieve a consistent design risk across the range in the input parameters and their variability. Based on extensive studies of the calibration process, and detailed evaluation of typical ranges in the key design parameters and the COV ranges that should be encountered for them, Phoon et al. (1995, 2003a,b) found that a 3×3 segmentation was sufficient for practical calibration. The typical ranges of COV to be expected have been presented in Table D-2 in Section D.2.

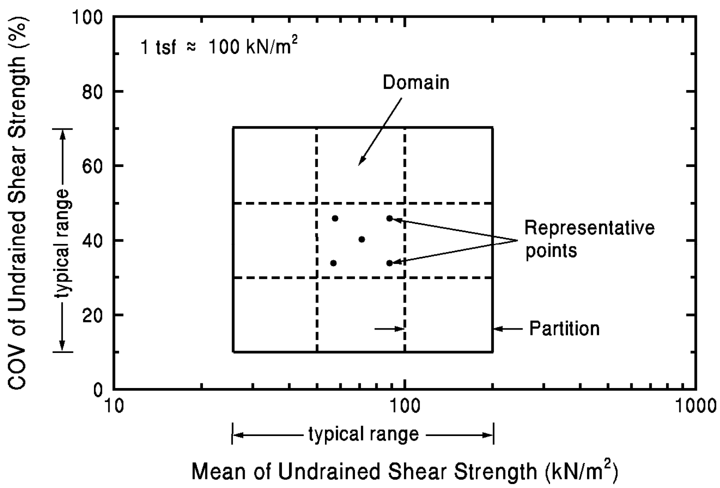

Figure D-3. Partitioning of parameter space for calibration of resistance factors (Phoon et al. 1995, Kulhawy et al. 2012)

D.5.4 Quantile-based design

It has been recognized that these code calibration methods can be unwieldy, because the COVs of design soil parameters and model factors are not constant and can vary over a wide range (Section D.2 and D.4), necessitating the segmentation of the calibration domain highlighted in Section D.5.3.

For geotechnical design problems with variable COVs, although it is not possible to maintain a uniform target reliability index with constant partial factors, it may be possible to maintain such uniformity with fixed quantiles (Ching & Phoon 2011). The quantile is calibrated rather than prescribed to achieve a specific target probability of failure. It decreases in a relatively unique way with the target probability of failure for a given performance function. It is intrinsically related to the performance function. Hence, the quantile for a given soil property, say s_u, will vary when the property is applied within the context of different performance functions, even for the same target probability of failure

D.5.5 Direct probability-based design method

Instead of using semi-probabilistic methods, such as partial factor method discussed in Section D.5.3, probability-based design (cf. Clause 7.6) can be directly applied to geotechnical practice, such as expanded RBD (Wang et al, 2011). In such cases, a large number of design alternatives are evaluated systematically, and the optimal design is chosen with the maximum utility and satisfying the acceptability requirement (i.e., requirement of failure probability pf being less than target probability of failure at various limit states).

The expanded RBD formulates the geotechnical design process (e.g., foundation design) as an expanded reliability problem in which a single run of Monte Carlo simulations (MCS) is used to address, explicitly and simultaneously, the ULS, SLS, economically-optimized limit state and reliability requirements of the design. An expanded reliability problem, as described herein, refers to a reliability analysis of a system in which a set of system design parameters are considered artificially as uncertain with probability distributions specified by the user for design exploration purposes. Design parameters, such as pile depth D and diameter B, are treated artificially as discrete uniform random variables, and the design process is considered as a process of finding failure probabilities for design alternatives with various combinations of B and D [i.e., conditional probability p(Failure|B,D)] and comparing them with a target probability of failure pT, which could be a ULS or SLS requirement.

D.5.6 System reliability

Because the majority of geotechnical structures have multiple failure modes, most geotechnical RBD is in fact system reliability problems (cf. Clause 7.3). For example, a simple gravity retaining wall has at least three failure modes: horizontal sliding along the base of the wall, overturning or rotation about the toe of the wall, and bearing capacity failure of the soil beneath the wall. These failure modes tend to interact among each other, because loads and capacities for different failure modes can be correlated. Consider, for example, self-weight of a gravity retaining wall, which is the major source of capacity against sliding and overturning failure modes, but at the same time, is also a major source of load for bearing capacity failure mode.

A pile foundation is a system of piles, which consists of several pile groups, each group consisting of a few individual piles. The evaluation of the reliability of the pile system requires the consideration of the reliability of the individual piles, the pile group effects, and the system effects arising from pile-superstructure interactions (Zhang et al. 2001).

Many geotechnical structures form failure mechanisms in the surrounding soil mass in the ultimate limit state (e.g. slopes, tunnels, deep excavations). Each potential slip surface in the soil mass is a failure mode. The probability of failure associated with the "most likely" failure mode identified by FORM only provides a lower bound for the system probability of failure. In contrast to a pile foundation where the sliding surface is mostly restricted to the interface between soil and pile, the trajectory of a slip surface in a soil mass is coupled to the specific realization of a random field and can only be determined through finite element analysis or comparable numerical methods. This class of system reliability problems is complex, because of the coupling between mechanics and spatial variability. However, it is not uncommon in geotechnical engineering.

D.5.7 Reliability target

As discussed in Clause 7.4, the target reliability index β_T (or the target failure probability p_T) should depend on the consequence and the nature of failure, the economic losses, the social inconvenience, and the amount of expense and effort required to reduce the probability of failure. Table D-12 summarizes target reliability indices recommended in existing geotechnical RBD codes (mostly foundation design codes). It is evident that the reliability target varies for different limit states and in different codes. It is worthy to note that a foundation often consists of many components such as individual piles. The levels of reliability of the entire foundation and the components are often not identical as discussed in D.5.6.

Table D-12. Summary of target reliability index in existing geotechnical RBD codes

Design Code	ULS Target Reliability Index	SLS Target Reliability Index
Electric Power Research Institute (EPRI) multiple resistance factor design (MRFD)	3.2	2.6
Canadian National Building Code (NCBC)	3.5	N/A
American Association of State Highway and Transportation Official (AASHTO) foundation design code	2.0 – 3.5	N/A
Eurocode 7	4.7	2.9

Note: N/A = not available; reference period of the reliability indices is 1 year; Eurocode 7 values refer to Reliability Class 2 (RC2)

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