Transformation Models and Multivariate Soil Databases

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1.1 Introduction

Transformation models (Phoon and Kulhway 1999) are valuable because they serve as "prior" information for correlation behaviors among various soil parameters. Useful compilations of these models are available in the literature (e.g., Djoenaidi 1985; Kulhawy and Mayne 1990; Mayne et al. 2001). For instance, it is common to estimate the friction angle (ϕ') of sand based on its SPT N value through a transformation model derived from data points obtained in the literature, as is showed in Figure 1-1. Here, the SPT N value is the site-specific information, and the friction angle ϕ' is the design soil parameter, also assumed as site-specific. However, the SPT N-ϕ' transformation model is not site-specific and is typically developed using a SPT N-ϕ' database collected from the literature. It is customary to adopt such a transformation model, which is not site-specific, in the process of estimating site-specific ϕ'. This process is illustrated in Figure 1-1. Suppose that the $(N_1)_{60}$ value (corrected N value) of a sand at certain depth at the design site is known to be 25. A vertical line is drawn at this $(N_1)_{60}$ value in Figure 1-1, and there are quite a few SPT N-ϕ' data points that are with similar $(N_1)_{60}$ values (circles). Although these data points are not site-specific, their ϕ' values may be meaningful. If the design site characteristics are within the coverage of the SPT N-ϕ' database, it is reasonable to think that the site-specific ϕ' value can be captured by the ensemble of these non-specific ϕ' values. By doing so, a single measurement of site-specific SPT N is converted into several "equivalent" ϕ' values that are viewed as posterior information for the actual site-specific ϕ' value.

Although this ensemble of "equivalent" ϕ' values (posterior information) may be a meaningful and realistic representation of the actual site-specific ϕ' value, there are occasion concerns expressed in the literature that the transformation models are applied too liberally in practice without careful consideration of their limitations. The purpose of this report is therefore to address the following practical questions:

1. What does the design soil parameter estimated from a transformation model really mean? The estimate can be a point estimate (e.g., the average of the equivalent ϕ' values) or an interval estimate (e.g., the range of the equivalent ϕ' values).
2. In what conditions will a transformation model produce meaningful estimates that are closely related to the actual site-specific design soil parameter?
3. In what conditions will a transformation model produce meaningless results that have very little to do with the actual site-specific design soil parameter?

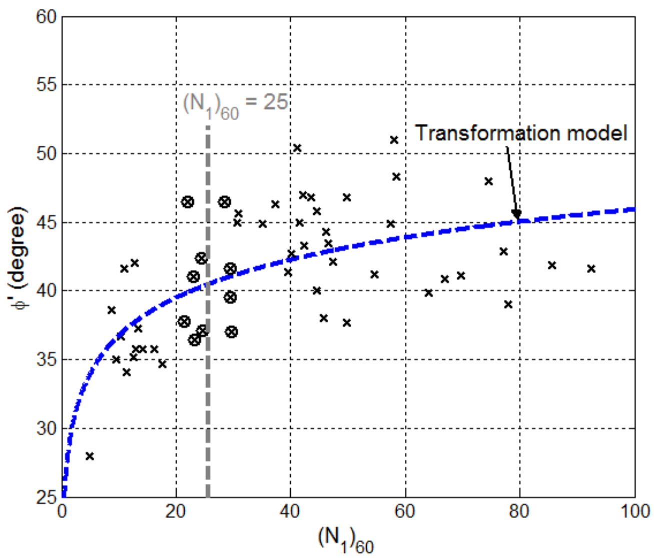

Figure 1-1 Transformation model between $(N_1)_{60}$ and ϕ' derived from data points in the literature.

This report will address the above questions through the "leave-one-out" design exercise based on a real soil database. The soil database is divided into two subsets: the first subset contains data points from a single site (design site), whereas the second subset contains the remaining sites in the database (training sites). The purpose of the leave-one-out exercise is to construct the transformation model based on the training sites, then estimate the design soil parameter for the design site. The effectiveness of the transformation model can then be verified by comparing the estimation result and the actual value of the design soil parameter. To understand the effect of adopting a "general" soil database versus a "regional" soil database, two clay databases are collected: one general database for generic clays, and one regional database for Finland clays.

1.2 Multivariate soil databases

Two clay databases and one sand database, shown in Table 1-1, are compiled. The databases are labelled as (soil type)/(number of parameters of interest)/(number of data points). The two clay databases will be adopted to conduct the leave-one-out design exercise. CLAY/10/7490 is a general clay database, whereas F-CLAY/7/216 is a reginal (Finland) clay database. SAND/7/2794 is a general sand database.

Table 1-1 Three multivariate soil databases.

Database	Reference	Parameters of interest	# data points	# sites/ studies	Range of properties		
					OCR	PI	S_t
CLAY/10/7490	Ching and Phoon (2014)	LL, PI, LI, σ'_v/P_a, σ'_p/P_a, s_u/σ'_v, S_t, $(q_t-\sigma_v)/\sigma'_v$, $(q_t-u_2)/\sigma'_v$, B_q	7490	251 studies	1~10	Low to very high plasticity	Insensitive to quick clays
F-CLAY/7/216	D'Ignazio et al. (2016)	s_u^{FV}, σ'_v, σ'_p, w_n, LL, PL, S_t	216	24 sites	1~7.5	Low to very high plasticity	Insensitive to quick clays
SAND/7/2794	Ching et al. (2016)	D_{50}, C_u, D_r, σ'_v/P_a, ϕ', q_{t1}, $(N_1)_{60}$	2794	176 studies	1~15	D_{50} = 0.1~40 mm C_u = 1~1000+ D_r = -0.1~117%	

Note: LL = liquid limit; PL = plastic limit; PI = plasticity index; LI = liquidity index; w_n = natural water content; D_{50} = median grain size; C_u = coefficient of uniformity; D_r = relative density; σ'_v = vertical effective stress; σ'_p = preconsolidation stress; s_u = undrained shear strength; s_u^{FV} = undrained shear strength from field vane; s_u^{re} = remoulded s_u; ϕ' = effective friction angle; S_t = sensitivity; OCR = overconsolidation ratio, $(q_t\text{-}\sigma_v)/\sigma'_v$ = normalized cone tip resistance; $(q_t\text{-}u_2)/\sigma'_v$ = effective cone tip resistance; u_0 = hydrostatic pore pressure; $(u_2\text{-}u_0)/\sigma'_v$ = normalized excess pore pressure; B_q = pore pressure ratio = $(u_2\text{-}u_0)/(q_t\text{-}\sigma_v)$; P_a = atmospheric pressure = 101.3 kPa; $q_{t1} = (q_t/P_a)\times C_N$ (C_N is the correction factor for overburden stress); $(N_1)_{60} = N_{60}\times C_N$ (N_{60} is the N value corrected for the energy ratio).

Table 1-2 Statistics for the CLAY/10/7490 database (Table 3 in Ching and Phoon 2014)

Variable	n*	Mean	COV*	Min	Max
LL	3822	67.7	0.80	18.1	515
PI	4265	39.7	1.08	1.9	363
LI	3661	1.01	0.78	-0.75	6.45
σ'_v/P_a	3370	1.80	1.47	4.13E-3	38.74
σ'_p/P_a	2028	4.37	2.31	0.094	193.30
s_u/σ'_v	3538	0.51	1.25	3.68E-3	7.78
S_t	1589	35.0	2.88	1	1467
B_q	1016	0.58	0.35	0.01	1.17
$(q_t\text{-}\sigma_v)/\sigma'_v$	862	8.90	1.17	0.48	95.98
$(q_t\text{-}u_2)/\sigma'_v$	668	5.34	1.37	0.61	108.20
s_u/σ'_p	1467	0.23	0.55	3.68E-3	1.34
OCR	3531	3.85	1.56	1.0	60.23
s_u^{re}/P_a	1143	0.075	2.86	9.67E-5	2.47

* n is the number of data points; COV stands for the coefficient of variation.

1.2.1 *CLAY/10/7490*

The CLAY/10/7490 database (Ching and Phoon 2014) is a general clay database consisting of data points from 251 studies. The geographical regions cover Australia, Austria, Brazil, Canada, China, England, Finland, France, Germany, Hong Kong, India, Iraq, Italy, Japan, Korea, Malaysia, Mexico, New Zealand, Norway, Northern Ireland, Poland, Singapore, South Africa, Spain, Sweden, Thailand, Taiwan, United Kingdom, United States, and Venezuela. The clay properties cover a wide range of overconsolidation ratio (OCR) (but mostly 1~10), a wide range of sensitivity (S_t) (sites with S_t = 1~ tens or hundreds are fairly typical), and a wide range of plasticity index (PI) (but mostly 8 ~ 100). Ten dimensionless parameters of clays are of primary interest: liquid limit (LL), plasticity index (PI), liquidity index (LI), normalized vertical effective stress (σ'_v/P_a) (P_a is one atmosphere pressure = 101.3 kN/m^2), normalized preconsolidation stress (σ'_p/P_a), normalized undrained shear strength (s_u/σ'_v) (s_u converted to the "mobilized" s_u defined by Mesri and Huvaj 2007), sensitivity (S_t), normalized piezocone tip resistance $(q_t-\sigma_v)/\sigma'_v$, and normalized effective piezocone tip resistance $(q_t-u_2)/\sigma'_v$, and piezocone pore pressure ratio B_q. Some other dimensionless parameters of interest, such as s_u/σ'_p, overconsolidation ratio (OCR), and s_u^{re}/P_a, can be derived from the above 10 parameters. The basic statistics of all these parameters (10 basic parameters together with s_u/σ'_p, OCR, and s_u^{re}/P_a) are listed in Table 1-2.

1.2.2 *F-CLAY/7/216*

The F-CLAY/7/216 database (D'Ignazio et al. 2016) is a regional clay database consisting of 216 field vane (FV) data points from 24 different test sites from Finland. Each data point contains genuine multivariate information on 7 clay parameters measured at comparable depths and sampling locations: FV undrained strength (s_u^{FV}), vertical effective stress (σ'_v), preconsolidation stress (σ'_p), water content (w), liquid limit (LL), plastic limit (PL), and sensitivity (S_t). The clay properties cover wide ranges of sensitivity S_t (2~64), plasticity PI (2~95), overconsolidation ratio OCR (1~7.5), and water content w (25~150). To be consistent with Table 1-2, these parameters are converted to dimensionless parameters PI, LI, σ'_v/P_a, σ'_p/P_a, s_u/σ'_v, etc., in which s_u = (design value of s_u) = $s_u^{FV}\times$(PI-dependent correction factor proposed by Bjerrum 1972). The basic statistics for these dimensionless parameters are listed in Table 1-3.

Table 1-3 Statistics for the F-CLAY/7/216 database

Variable	n	Mean	COV	Min	Max
LL	216	66.3	0.30	22.0	125.0
PI	216	38.5	0.48	2.0	95.0
LI	216	1.44	0.46	0.42	4.80
σ'_v/P_a	216	0.46	0.48	0.074	1.61
σ'_p/P_a	216	0.79	0.50	0.20	2.27
s_u/σ'_v	216	0.40	0.74	0.11	2.71
S_t	216	17.4	0.79	2	64
s_u/σ'_p	216	0.22	0.31	0.058	0.52
OCR	216	1.84	0.51	1.0	7.5
s_u^{re}/P_a	216	0.016	0.99	0.0011	0.14

1.2.3 *Comparison between CLAY/10/7490 and F-CLAY/7/216*

The main difference between the two databases is that CLAY/10/7490 is a general database, whereas F-CLAY/7/216 is a regional database. A preliminary comparison between Tables 1-2 and 1-3 indicates the following distinct features between the general and regional databases:

1. The number of data points for the general database is significantly larger than that for the regional database.
2. The range spanned between the minimum and maximum values for the general database is significantly wider than that for the regional database. As a result, the COVs for the general database are significantly larger than those for the regional database.

Figure 1-2 shows the LI-S_t, OCR-s_u/σ'_v, LI-σ'_p/P_a, and PI-s_u/σ'_p relationships for the two databases. It is clear that the coverage of the general database (CLAY/10/7490) is wider than the coverage of the regional database (F-CLAY/7/216). There are six data points from four Finland sites (annotated in Figure 1-2) with LI > 3, but with exceptionally low S_t. These six data points in F-CLAY/7/216 are not within the coverage of the general database CLAY/10/7490.

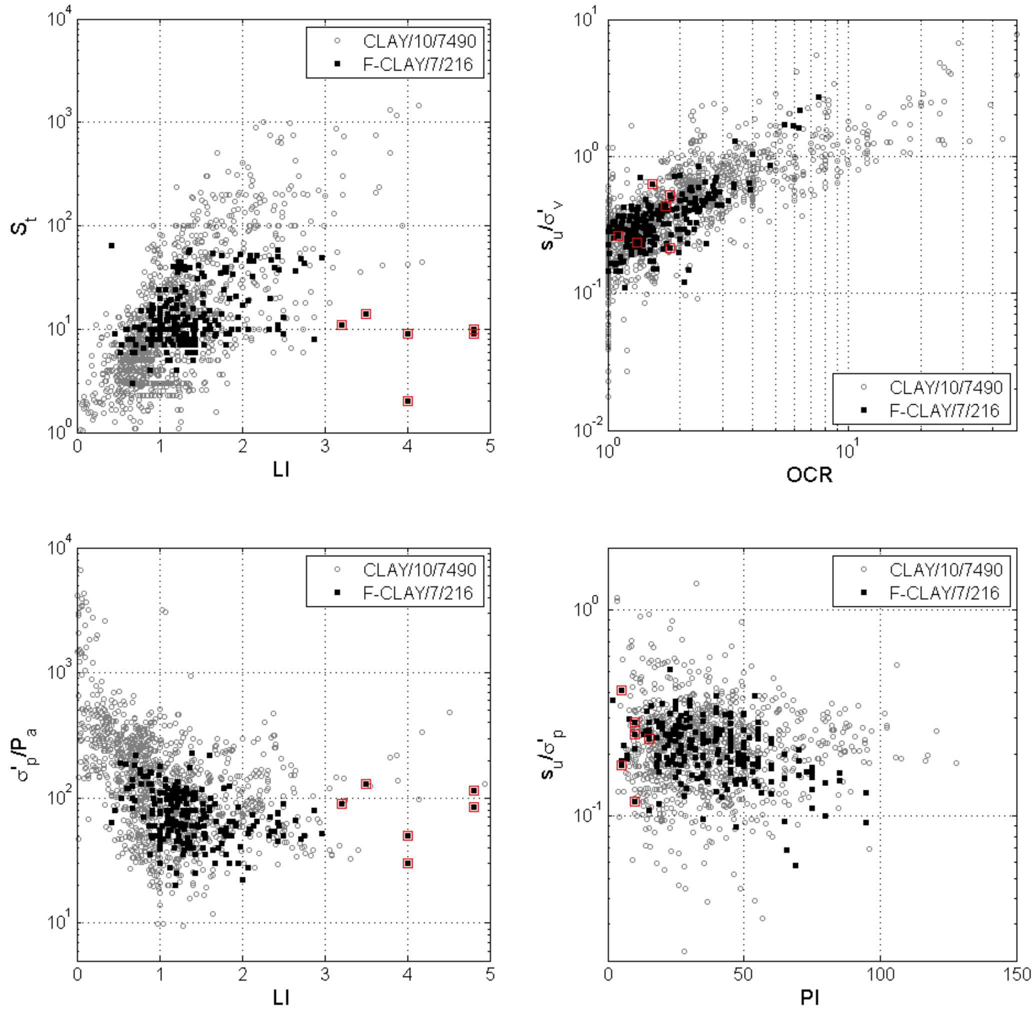

Figure 1-2 LI-S_t, OCR-s_u/σ'_v, LI-σ'_p/P_a, and PI-s_u/σ'_p relationships for the two databases.

1.3 Use of database in estimating site-specific design soil parameter

In the design process, a soil database can be adopted to develop a transformation model that can be further used to estimate the design soil parameter (e.g., s_u) for the design site based on site-specific information. For instance, based on the site-specific OCR information of a clay at a design site, its s_u/σ'_v value can be estimated from an OCR-s_u/σ'_v transformation model developed from a clay database. Note that the clay database and the resulting transformation model are not site-specific. A question may arise: is the resulting s_u/σ'_v estimate site-specific or not? This question can be answered by comparing the s_u/σ'_v estimate with the actual site-specific s_u/σ'_v value. If the s_u/σ'_v estimate can capture the actual site-specific s_u/σ'_v value, the s_u/σ'_v estimate is site-specific. Otherwise, it is not. This comparison can be realized by the leave-one-out design exercise. The details for this leave-one-out exercise will be presented later. Consider two scenarios:

1. Scenario 1: A regional database, such as F-CLAY/7/216, is adopted to develop the OCR-s_u/σ'_v transformation model.
2. Scenario 2: A regional database is not available. A general database, such as CLAY/10/7490, is used to develop the OCR-s_u/σ'_v transformation model.

For both scenarios, the question whether the resulting s_u/σ'_v estimate is site-specific or not will be addressed. The effect for adopting a regional database against a general database will be also illustrated.

1.3.1 *Scenario 1*

Let us consider a new Finland site, and suppose the design engineer has the regional database F-CLAY/7/216. Consider a clay at that site with a known site-specific OCR, denoted by OCR_{new}. The goal is to estimate its site-specific s_u/σ'_v, denoted by $(s_u/\sigma'_v)_{new}$. The estimate can be either a point estimate or an interval estimate. The design engineer can adopt the OCR-s_u/σ'_v data points in the database to develop the following transformation model:

$$\ln\left(s_u/\sigma'_v\right) = a + b \times \ln\left(OCR\right) + \varepsilon \tag{1-1}$$

where (a, b) are unknown coefficients to be estimated; ε is the transformation error, modeled as a zero-mean normal random variable with standard deviation σ_ε. (a, b) can be estimated by least squares:

$$\begin{aligned}
&b^* = S_{xy}/S_{xx} \qquad a^* = y_m - b^* \times x_m \\
&x_m = \frac{1}{n}\sum_{i=1}^{n}\log\left(OCR_i\right) \qquad y_m = \frac{1}{n}\sum_{i=1}^{n}\log\left[\left(s_u/\sigma'_v\right)_i\right] \\
&S_{xx} = \sum_{i=1}^{n}\left[\log\left(OCR_i\right) - x_m\right]^2 \qquad S_{xy} = \sum_{i=1}^{n}\left[\log\left(OCR_i\right) - x_m\right] \times \left(\log\left[\left(s_u/\sigma'_v\right)_i\right] - y_m\right)
\end{aligned} \tag{1-2}$$

where (a^*, b^*) denote the least square estimates for (a, b); OCR_i and $(s_u/\sigma'_v)_i$ denote the OCR and (s_u/σ'_v) values of the i-th data point is the clay database; n is the number of data points in the database. σ_ε can be estimated as well:

$$\sigma_\varepsilon^* = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}\left\{\log\left[\left(s_u/\sigma'_v\right)_i\right] - a^* - b^* \times \log\left(OCR_i\right)\right\}^2} \tag{1-3}$$

The design engineer can then obtain two useful estimates for $\ln[(s_u/\sigma'_v)_{new}]$: (a) the point estimate $a^*+b^*\times\ln(OCR_{new})$; and (b) the 95% confidence interval (CI) estimate defined by:

$$a^* + b^* \times \ln\left(OCR_{new}\right) \ \pm \ t_{0.975} \times \sigma_\varepsilon^* \times \sqrt{1+\frac{1}{n}+\frac{\left[\ln\left(OCR_{new}\right)-x_m\right]^2}{S_{xx}}} \tag{1-4}$$

where $t_{0.975}$ is the 97.5% percentile for the Student-t distribution with (n-2) degrees of freedom. In the following illustration, we will focus on the 95% confidence interval (CI) estimate. The 95% CI in Eq. (1-4) is a "nominal" 95% CI. It is unclear whether it is a genuine 95% CI. Namely, it is unclear whether the actual chance for $\ln[(s_u/\sigma'_v)_{new}]$ to be within the interval is indeed close to 95%.

To illustrate that the nominal 95% CI is genuine, consider the following "leave-one-out" design exercise. There are 24 sites in the F-CLAY/7/216 database. Each time, one site is treated as the new design site, whereas the remaining 23 sites are treated as the training sites. Note that the new design site and the training sites belong to the same "population": they are all Finland sites. However, the design site is independent of the 23 training sites. There may be several clay data points in the design site. For each clay at the design site, its OCR_{new} is considered known, e.g., an oedometer test is conducted to determine its OCR. However, we "pretend" its $(s_u/\sigma'_v)_{new}$ to be unknown. First, $(a^*,b^*,\sigma_\varepsilon^*)$ are estimated based on the 23 training sites using Eqs. (1-2) and (1-3). The point estimate for $\ln[(s_u/\sigma'_v)_{new}]$ is $a^*+b^*\times \ln(OCR_{new})$ and the nominal 95% CI for $\ln[(s_u/\sigma'_v)_{new}]$ is obtained using Eq. (1-4). Because $\ln[(s_u/\sigma'_v)_{new}]$ for the clay is actually known, we can compute the prediction error $e = \ln[(s_u/\sigma'_v)_{new}]-a^*-b^*\times\ln(OCR_{new})$ and also determine whether $\ln[(s_u/\sigma'_v)_{new}]$ is within the nominal 95% CI. This leave-one-out exercise is repeated for all 216 data points in F-CLAY/7/216. Figure 1-3 shows the histogram of the 216 prediction errors. The prediction errors have a mean value that is roughly zero. Among the 216 leave-one-out trials, $\ln[(s_u/\sigma'_v)_{new}]$ is within the nominal 95% CI for 202 times. This means that the CI is in effect a 202/216 = 93.5% CI: it is reasonably close to a genuine 95% CI. The difference between 93.5% and 95% may be partially due to the statistical error.

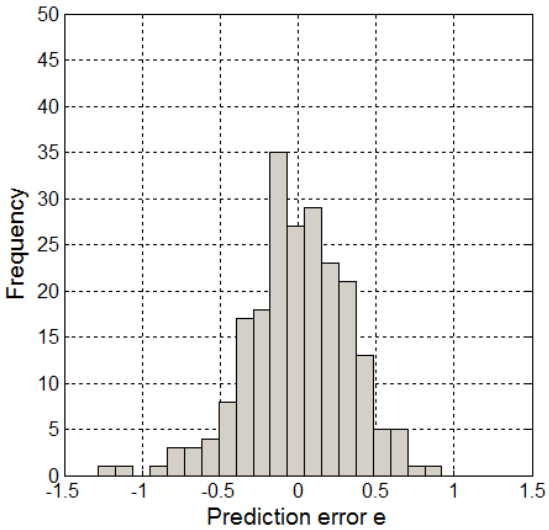

Figure 1-3 Histogram of the prediction error e = $\ln[(s_u/\sigma'_v)_{new}] - a^* - b^* \times \ln(OCR_{new})$.

The above leave-one-out design exercise shows that the nominal 95% CI developed by the 23 training sites is close to genuine. This is probably because the design site and the 23 training sites belong to the same population, e.g., Finland sites. When this happens (same population), the nominal 95% CI is theoretically the genuine 95% CI. This conclusion will not change if all 24 sites in F-CLAY/7/216 are adopted to develop the transformation model and goal is to estimate the $(s_u/\sigma'_v)_{new}$ for a 25th site that is not in F-CLAY/7/216. This justifies the use of a transformation model developed by a regional database: if the 25th site is within the same region (i.e., same population), the resulting nominal 95% CI for the $\ln[(s_u/\sigma'_v)_{new}]$ of this 25th site will be close to genuine. That is to say, the chance for $\ln[(s_u/\sigma'_v)_{new}]$ to be within the 95% CI will be close to 95%.

There is a caveat here: the soil database needs to have a sufficient coverage to represent the entire region, i.e., the population. In the above leave-one-out design exercise, there are 24 – 1 = 23 training sites. If the number of sites is small, the training sites can no longer represent the Finland population, and the nominal 95% CI can cease to be genuine, so the actual chance for $\ln[(s_u/\sigma'_v)_{new}]$ to be within the nominal 95% CI will not be close to 95%. Figure 1-4 shows how this actual chance varies with respect to the number of training sites (see the line with legend "Scenario 1"). Consider a subset database with (n_t + 1) sites randomly sampled from the 24 sites in F-CLAY/7/216. The leave-one-out design exercise is conducted on the subset database with n_t training sites and one design site, and the chance for $\ln[(s_u/\sigma'_v)_{new}]$ to be within the 95% CI can be evaluated. This actual chance is itself random because it depends on the random sampling effect of the (n_t + 1) sites. Therefore, the subset database with (n_t + 1) sites is randomly sampled for 100 times to obtain 100 samples for the actual chance. The horizontal axis in Figure 1-4 is the number of training sites (n_t) and the vertical axis is the average of the 100 samples for the actual chance. The (averaged) actual chance seems to converge to 95% with increasing number of sites. The actual chance is significantly less than 95% if the database contains less than 4 Finland sites, whereas the actual chance is close to 95% if the database contains more than 10 Finland sites. This means that for the Finland case, a regional database with more than 10 sites should have a sufficient coverage.

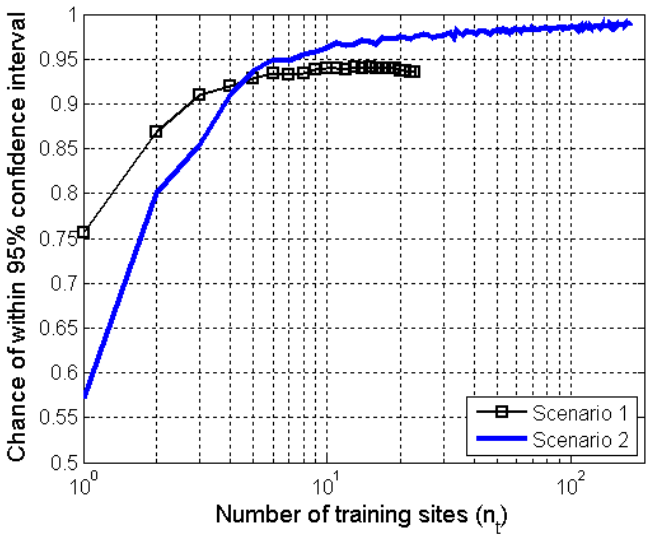

Figure 1-4 Chance for $\ln[(s_u/\sigma'_v)_{new}]$ to be within the nominal 95% CI.

1.3.2 Scenario 2

Suppose that the new design site is a Finland site, but a Finland database is not available. Yet, suppose the design engineer has the general database CLAY/10/7490. Note here that now the design site and the training sites do not belong to the same population: the training sites obviously have a wider coverage because they are global sites. The OCR-s_u/σ'_v data points in CLAY/10/7490 are from 179 global sites from Americas, Europe, Asia, etc. The design engineer can still adopt the OCR-s_u/σ'_v transformation model developed from CLAY/10/7490 to obtain the nominal 95% CI, but is it still a genuine 95% CI with respect to the Finland design site?

To understand the significance of the nominal 95% CI obtained from a general database, the following design exercise is taken. It is not necessary to do leave-one-out, because the design site is not within the general database. First, (a^*, b^*, σ_ε^*) are estimated using Eqs. (1-2) and (1-3) based on the general database. Each clay data point in F-CLAY/7/216 is a Finland design case. Its OCR value is treated as known (denoted by OCR_{new}), whereas we pretend its s_u/σ'_v value to be unknown [denoted by $(s_u/\sigma'_v)_{new}$]. The nominal 95% CI for this $\ln[(s_u/\sigma'_v)_{new}]$ can be obtained using Eq. (1-4) based on (a^*, b^*, σ_ε^*) and OCR_{new}. Nonetheless, $\ln[(s_u/\sigma'_v)_{new}]$ is actually known, and we can determine whether $\ln[(s_u/\sigma'_v)_{new}]$ is within the nominal 95% CI.

This exercise is repeated for all 216 data points in F-CLAY/7/216. It turns out that the actual chance for $\ln[(s_u/\sigma'_v)_{new}]$ of a Finland clay to be within the nominal 95% CI is 99.1%, significantly larger than 95%. This is probably because the design site and the 179 training sites belong to different populations. When this happens (different populations), there is no guarantee that the nominal 95% CI is genuine.

There are 179 sites in the general database CLAY/10/7490. It is interesting to know that the actual chance will change if there is a different number of training sites. Figure 1-4 shows how the actual chance varies with respect to the number of training sites (n_t) (see the line with legend "Scenario 2"). Again, the actual chance is random due to the random sampling effect of the n_t sites. Therefore, the subset database with n_t sites is randomly sampled from the 179 sites for 100 times to obtain 100 samples for the actual chance. The actual chance seems to converge to 100% with increasing number of training sites, rather than converge to 95%. The 95% CI developed from a general database with many sites is "conservative" with respect to a Finland site, in the sense that the actual chance for $\ln[(s_u/\sigma'_v)_{new}]$ of a Finland site to be within the 95% CI is more than 95%. The nominal 95% CI is wider than the genuine 95% CI, because CLAY/10/7490 has a wider coverage than the Finland database F-CLAY/7/216. This wider coverage can be clearly seen in Figure 1-2. Nonetheless, the actual chance can be significantly less than 95% if the general database contains less than 4 sites.

1.3.3 Scenario "A"

Let us consider a rather academic scenario: the new design site belongs to the "general population": the population containing all global sites. We call this scenario "Scenario A". The purpose of this scenario is to further verify the significance of the nominal 95% CI developed from the general database. Let a clay at the design site have a known site-specific OCR, denoted by OCR_{new}. The goal is to estimate its unknown s_u/σ'_v, denoted by $(s_u/\sigma'_v)_{new}$. The nominal 95% CI for $\ln[(s_u/\sigma'_v)_{new}]$ is constructed by the general database CLAY/10/7490. Note that the sites in the general database CLAY/10/7490 also belong to the general population. Therefore, the design site and the training sites belong to the same general population. Is this nominal 95% CI the genuine 95% CI with respect to the new design site?

The following leave-one-out design exercise is adopted to illustrate the significance of the nominal 95% CI. The OCR-s_u/σ'_v data points in the CLAY/10/7490 database are from 179 sites. Each time, one site is treated as the design site, whereas the remaining 178 sites are treated as the training sites. Note that the design site and the 178 training sites belong to the same general population. First, (a^*, b^*, σ_ε^*) are estimated based on the 178 training sites using Eqs. (1-2) and (1-3), and the nominal 95% CI for $\ln[(s_u/\sigma'_v)_{new}]$ is obtained using Eq. (1-4). Nonetheless, the $\ln[(s_u/\sigma'_v)_{new}]$ for the design site is actually known so that we can determine the chance for $\ln[(s_u/\sigma'_v)_{new}]$ to be within the nominal 95% CI. The actual chance for

$\ln[(s_u/\sigma'_v)_{new}]$ to be within the nominal 95% CI is about 94.4%, close to 95%. Again, the caveat is that there is a sufficient number of sites in the database. Figure 1-5 shows how the actual chance varies with respect to the number of training sites (n_t). Again, this actual chance is random because it depends on the random sampling effect of the training sites. The vertical axis is the average of the 100 samples for the actual chance. The (averaged) actual chance seems to converge to 95% with increasing number of sites. The actual chance is close to 95% if the number of sites is more than 50-100.

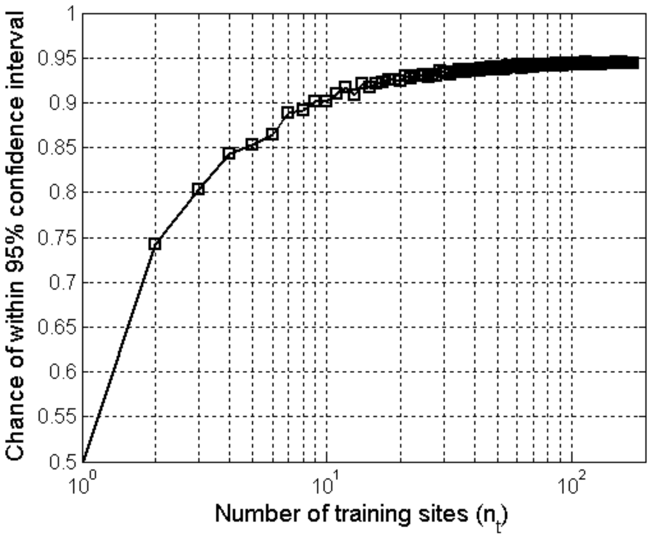

Figure 1-5 Chance for $\ln[(s_u/\sigma'_v)_{new}]$ to be within the nominal 95% CI (Scenario A).

1.3.4 Discussions

The key questions that this study aims to address are:

1. What does the design soil parameter estimate from a transformation model really mean?
2. In what conditions will a transformation model produce meaningful estimates that are closely related to the actual site-specific design soil parameter?

Based on the above results, it can be concluded that the nominal 95% CI produced by the transformation model is meaningful because it has a large chance to include the actual site-specific $\ln[(s_u/\sigma'_v)_{new}]$. Moreover, the nominal 95% CI is close to genuine, as long as the design site and the training sites belong to the same population. The following two scenarios exemplify the concept of "same population":

1. The design site is a Finland site, whereas the soil database is a Finland (regional) soil database that has a sufficient coverage. This is Scenario 1. For the Finland case, 10 sites in the regional database seem sufficient.
2. The design site belongs to the general population, whereas the soil database is a

general database with a sufficient coverage. This is Scenario A. In the above illustration, 50-100 sites in the general database seem sufficient.

Although the nominal 95% CI provides a satisfactory estimate for the site-specific $\ln[(s_u/\sigma'_v)_{new}]$, it is an interval estimate, not a point estimate. It is possible to obtain the point estimate, i.e., the point estimate = $a^*+b^*\times\ln(OCR_{new})$, but certain inaccuracy is to be expected (see the prediction error in Figure 1-3).

3. In what conditions will a transformation model produce meaningless results that have little to do with the actual site-specific design soil parameter?

If the design site and the training sites do not belong to the same population, there is no guarantee that the nominal 95% CI derived from the training sites is genuine. If the design site belongs to the Finland population but the training sites are general with a sufficient number of sites, the nominal 95% confidence interval derived from the general database will be wider the genuine 95% CI. When this happens, the nominal 95% CI is still meaningful (because it still has a large chance to include the actual $\ln[(s_u/\sigma'_v)_{new}]$) but less effective.

The nominal 95% CI may become completely meaningless if the design site and training sites belong to two populations occupying completely different regions in the OCR-(s_u/σ'_v) space. For instance, the design site contains fissured clays, whereas the training sites only contains non-fissured clays.

Appendix A (Transformation Models Calibrated by Soil Databases) shows some transformation models calibrated by the F-CLAY/7/216 regional database and by the CLAY/10/7490 general database. These transformation models were originally developed in the literature, but their biases and variabilities are calibrated by the soil databases. Given the site-specific investigation information of a new design site, the point estimate and nominal 95% CI can be obtained from these transformation models (details given in Appendix A). The 95% CI estimate is meaningful in the sense that the actual design soil parameter will have a large chance to be within the confidence interval. Appendix A also shows some transformation models for sands as well as their biases and variabilities calibrated by the SAND/7/2794 general database.

1.3.5 Other transformation models

For other transformation models, the qualitative conclusions obtained above remain unchanged. Consider the LI-S_t transformation model. Figure 1-6 shows how the actual chance for $\ln[(S_t)_{new}]$ to be within the nominal 95% CI varies with the number of sites in the database. The left plot is for Scenarios 1 and 2, whereas the right plot is for Scenario A. Those plots are qualitatively similar to Figures 1-4 and 1-5.

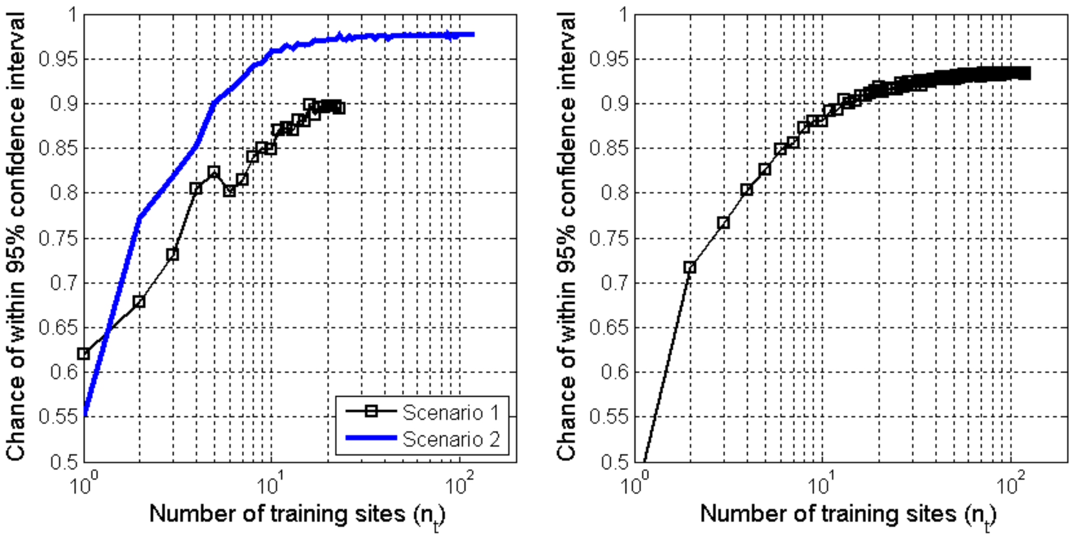

Figure 1-6 Chance for $\ln[(S_t)_{new}]$ to be within the nominal 95% CI: (left) Scenarios 1 & 2; (right) Scenario A.

1.3.6 Multivariate correlations

We have illustrated how $\ln[(s_u/\sigma'_v)_{new}]$ can be estimated based on the site-specific OCR_{new}. It was shown that the nominal 95% CI developed from a soil database can be useful and meaningful. However, it can happen that the resulting 95% CI is very wide so that $\ln[(s_u/\sigma'_v)_{new}]$ is still very uncertain. Multivariate information is usually available in a typical site investigation. For instance, when undisturbed samples are extracted for oedometer tests to determine OCR, piezocone test (CPTU) may be conducted in close proximity. These multiple data sources are typically correlated to the design soil parameter, e.g., the undrained shear strength (s_u). Figure 1-7 shows the data points for the two transformations in the CLAY/10/7490 database. It is clear that both OCR and $(q_t-\sigma'_v)/\sigma'_v$ are positively correlated to s_u/σ'_v. These multiple correlations can be exploited to reduce the uncertainty in the design soil parameter. In the previous sections, we have illustrated a framework where the site-specific OCR information can be used to obtain the 95% CI for (s_u/σ'_v). This univariate framework is extended to account for multivariate framework, e.g., both OCR and $(q_t-\sigma'_v)/\sigma'_v$ are known, in the following.

Suppose that a design engineer has a multivariate OCR-$[(q_t-\sigma'_v)/\sigma'_v]$-$(s_u/\sigma'_v)$ database. For each data point, OCR, $(q_t-\sigma'_v)/\sigma'_v$, and s_u/σ'_v are simultaneously known. The engineer can adopt the data points in the database to develop the following multivariate transformation model:

$$\ln\left(s_u/\sigma'_v\right) = a + b\times\ln\left(OCR\right) + c\times\ln\left[\left(q_t-\sigma_v\right)/\sigma'_v\right] + \varepsilon \qquad (1\text{-}5)$$

where (a, b, c) are unknown coefficients to be estimated; ε is the transformation error, modeled as a zero-mean normal random variable with standard deviation σ_ε. (a, b, c) can be estimated by least squares:

$$
\begin{bmatrix} a^* \\ b^* \\ c^* \end{bmatrix} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{y}
$$

$$
\mathbf{A} = \begin{bmatrix} 1 & \log(OCR_1) & \log\left(\left[(q_t-\sigma_v)/\sigma'_v\right]_1\right) \\ 1 & \log(OCR_2) & \log\left(\left[(q_t-\sigma_v)/\sigma'_v\right]_2\right) \\ \vdots & \vdots & \vdots \\ 1 & \log(OCR_n) & \log\left(\left[(q_t-\sigma_v)/\sigma'_v\right]_n\right) \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} \log\left[(s_u/\sigma'_v)_1\right] \\ \log\left[(s_u/\sigma'_v)_2\right] \\ \vdots \\ \log\left[(s_u/\sigma'_v)_n\right] \end{bmatrix} \tag{1-6}
$$

σ_ε can be estimated as well:

$$
\sigma_\varepsilon^* = \sqrt{\frac{1}{n-3}\sum_{i=1}^{n}\left\{\log\left[(s_u/\sigma'_v)_i\right] - a^* - b^*\times\log(OCR_i) - c^*\times\log\left(\left[(q_t-\sigma_v)/\sigma'_v\right]_i\right)\right\}^2} \tag{1-7}
$$

Based on the OCR-$[(q_t-\sigma'_v)/\sigma'_v]$-$(s_u/\sigma'_v)$ data points in CLAY/10/7490, the estimated σ_ε^* is equal to 0.46. With the OCR-(s_u/σ'_v) information from the same data points, the estimated σ_ε^* for the univariate OCR-(s_u/σ'_v) transformation model in Eq. (1-1) is equal to 0.51. This shows that the transformation uncertainty in the multivariate model (Eq. 1-5) is less than that in the univariate model (Eq. 1). The resulting 95% CI for $\ln[(s_u/\sigma'_v)_{new}]$ from the multivariate model (to be presented below) will be also narrower than that from the univariate model.

Now consider a new design site with known site-specific OCR and $(q_t-\sigma_v)/\sigma'_v$, denoted by OCR_{new} and $[(q_t-\sigma_v)/\sigma'_v]_{new}$. The goal is to estimate its site-specific s_u/σ'_v, denoted by $(s_u/\sigma'_v)_{new}$. Based on (a^*, b^*, c^*, σ_ε^*), the design engineer can obtain two useful estimates for $\ln[(s_u/\sigma'_v)_{new}]$: (a) the point estimate $a^*+b^*\times\ln(OCR_{new})+c^*\times\ln([(q_t-\sigma_v)/\sigma'_v]_{new})$ and (b) the nominal 95% CI estimate defined by:

$$
a^* + b^*\times\ln(OCR_{new}) + c^*\times\log\left(\left[(q_t-\sigma_v)/\sigma'_v\right]_{new}\right) \;\pm\; t_{0.975}\times\sigma_\varepsilon^*\times\sqrt{1+\mathbf{A}_{new}^T\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}_{new}} \tag{1-8}
$$

where $t_{0.975}$ is the 97.5% percentile for the Student-t distribution with (n-3) degrees of freedom; $\mathbf{A}_{new} = [1\ \ln(OCR_{new})\ \ln([(q_t-\sigma_v)/\sigma'_v]_{new})]^T$.

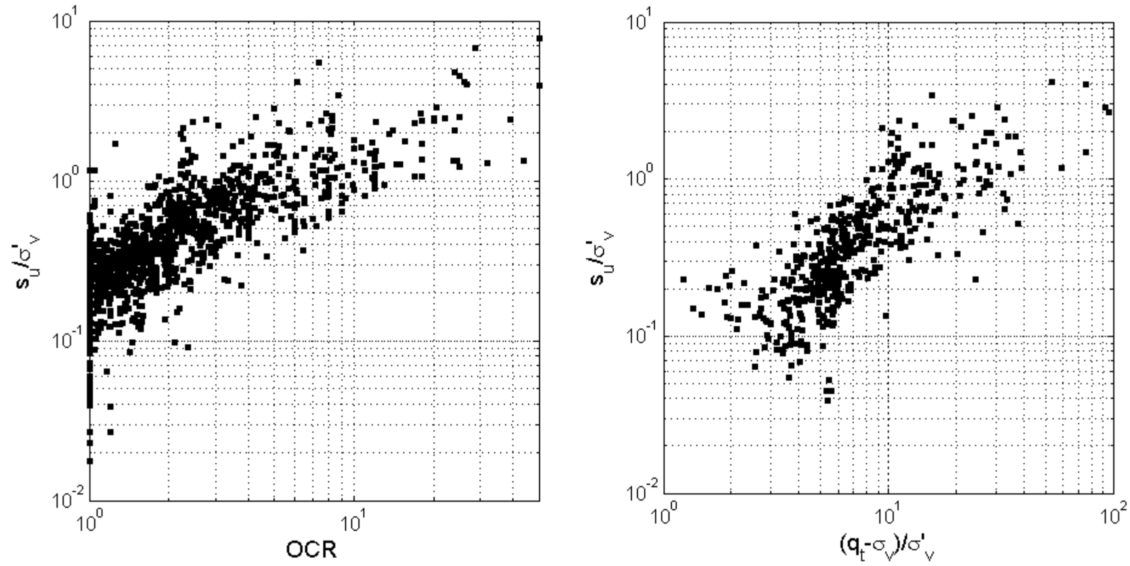

Figure 1-7 OCR-(s_u/σ'_v) and [(q_t-σ'_v)/σ'_v]-(s_u/σ'_v) data points in the CLAY/10/7490 database.

The following leave-one-out exercise based on CLAY/10/7490 is adopted to verify whether the nominal 95% CI is genuine. There are 50 sites in the CLAY/10/7490 database containing 417 multivariate OCR-[(q_t-σ'_v)/σ'_v]-(s_u/σ'_v) data points. Each time, one site is treated as the design site, whereas the remaining 49 sites are treated as the training sites that are further used to obtain the nominal 95% CI. This is similar to Scenario A above. For the leave-one-out exercise, ln[$(s_u/\sigma'_v)_{new}$] is actually known. Therefore, we can determine whether ln[$(s_u/\sigma'_v)_{new}$] is within the nominal 95% CI. Among the 417 leave-one-out trials, ln[$(s_u/\sigma'_v)_{new}$] is within the nominal 95% CI for 381 times. This means that the CI is in effect a 381/417 = 91.4% CI. The difference between 91.4% and 95% may be partially due to the statistical error. It is also possible that 50 sites are not yet sufficient for the convergence. Figure 1-8 shows how the actual chance for ln[$(s_u/\sigma'_v)_{new}$] to be within the nominal 95% CI varies with respect to the number of sites in the database. The convergence behavior in this figure is similar to those in Figures 1-4 to 1-6. It is possible that the qualitative conclusions obtained for the univariate framework above still apply to the multivariate framework.

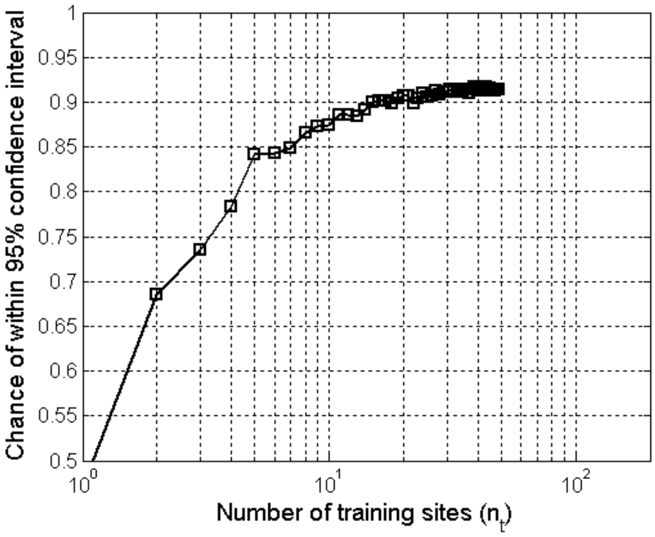

Figure 1-8 Chance for $\ln[(s_u/\sigma'_v)_{new}]$ to be within the nominal 95% CI (multivariate scenario).

1.4 Conclusions

A transformation model is frequently used to estimate the design soil parameter. However, it is not clear what the estimated design soil parameter really means. A possible concern for such a soil parameter estimate is that a transformation model is constructed by non-site-specific data points. Can these non-site-specific data points be used to derive any meaningful site-specific estimate? The purpose of this report is to address this question and to verify the significance of this design soil parameter estimate based on the so-called "leave-one-out" design exercise.

The leave-one-out exercise emulates the process of estimating the design soil parameter: the design soil parameter for a "design site" is estimated based on the transformation model constructed by a set of "training sites". The design site is not within the training sites. Basically, a large soil database with N sites is divided into two subsets: one subset only contains the design site, and the other subset contains N-1 training sites. A transformation model is first calibrated by the training sites, then it is adopted to estimate the design soil parameter for the design site. This process is repeated for all data points in the soil database. Because the actual value of the design soil parameter for the design site is in fact known, the performance and significance of the design soil parameter estimate obtained from the transformation model can be verified. In this report, we focus on the 95% confidence interval (CI) estimate obtained from the transformation model. This 95% CI may or may not be the genuine 95% CI, so it is called, in this report, the "nominal" 95% CI.

The results show that the nominal 95% CI estimate obtained from the transformation model is meaningful, albeit the transformation model is derived from non-site-specific data points. The concept of "population" in statistics is central to our conclusions. It is concluded that as long as the design site and training sites belong to the same population, the nominal 95% CI estimate obtained from the transformation model is close to a genuine 95% CI, meaning that the chance for the actual design parameter to be within the nominal 95% CI is close to 95%. A radical view is that only the data points at the design site (site-specific data points) can be used to derive the design soil parameter and that all non-site-specific data points are irrelevant. Nonetheless, the findings in this report suggest that this view may be incorrect. In fact, non-site-specific data points can be still useful if they are in the same "population" for the design site. This means that if the design site is a Finland site, the transformation model developed by Finland training sites (i.e., a Finland database) can be useful and meaningful in the sense that the resulting nominal 95% CI is close to a genuine 95% CI.

A more controversial scenario is that the design site and training sites do not belong to the same population, e.g., the design site is a Finland site, yet the training sites are general (global) sites. In the case that the design site population is a subset of the training site population (e.g., the Finland population is a subset of the general population), the results in this report suggest that the resulting nominal 95% CI is no longer a genuine 95% CI. Moreover, the nominal 95% CI is wider than the genuine 95% CI. In one previous illustration in this report (Scenario 2), the chance for the actual design parameter to be within the nominal 95% CI is close to 95% is 99.1%. Yet, this does not suggest that the nominal 95% CI is completely useless and meaningless. Instead, this only suggests that the nominal 95% CI is less effective and more conservative.

An even worse scenario is that the design site and training sites not only belong to different populations but also the design site population is not a subset of the training site population. For instance, the design site is with fissured clays, yet the training sites do not contain fissured clays. When this occurs, the resulting nominal 95% CI can become useless and meaningless.

Appendix A shows some transformation models calibrated by some soil databases. The guideline for deriving the point estimate and 95% CI estimate is also provided.

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Appendix 1A: Transformation models calibrated by soil databases

This appendix presents the calibration results for some transformation models in the literature. The calibrated models can be used to develop the point estimate and 95% confidence interval (CI) for the design soil parameter. The bias and variability for the clay transformation models are calibrated by the F-CLAY/7/216 and CLAY/10/7490 databases (see Table 1A-1), whereas the sand transformation models are calibrated by the SAND/7/2794 database (see Table 1A-2).

To explain the significance of the bias and variability for a transformation model, consider the first model in Table 1A-1, the LI-(s_u^{re}/P_a) model proposed by Locat and Demers (1988). The actual target value is s_u^{re}/P_a, and the predicted target value is $0.0144\times LI^{-2.44}$. For each data point in the database with simultaneous knowledge of (LI, s_u^{re}), (actual target value)/(predicted target value) = $(s_u^{re}/P_a)/(0.0144\times LI^{-2.44})$ can be computed. The sample mean of this ratio is called the bias factor (b) for the transformation model. The sample coefficient of variation (COV) of this ratio is called the COV (δ) of the transformation model. To be specific,

$$\text{Actual target value} = \text{predicted target value} \times b \times \varepsilon \qquad \text{(1A-1)}$$

where b is the bias factor (b = 1 means unbiased), and ε is the variability term with mean = 1 and COV = δ. If $\delta = 0$, there is no data scatter about the transformation model, i.e. the prediction is single-valued or deterministic, rather than a distribution. The calibrated bias factors and COVs for all clays and sand transformation models are shown in the last two columns of Tables 1A-1 and 1A-2, respectively. The number of data points used for each calibration is listed in the table ('n' in the third column).

The calibrated bias and COV of a transformation model can be adopted to develop the point estimate and 95% CI, described as follows. Consider again the LI-(s_u^{re}/P_a) model, let the site-specific LI value for the new design site be denoted by LI_{new}, the point estimate for $(s_u^{re}/P_a)_{new}$ is simply b×(predicted target value) = $b\times(0.0144\times LI_{new}^{-2.44})$. By assuming ε to be lognormal, the 95% CI for $(s_u^{re}/P_a)_{new}$ can be expressed as

$$\begin{aligned} &\frac{b\times\left(\text{predicted target value}\right)}{\sqrt{1+\delta^2}}\times\exp\left(\pm1.96\times\sqrt{\ln\left(1+\delta^2\right)}\right) \\ &=\frac{b\times\left(0.0144\times LI_{new}^{-2.44}\right)}{\sqrt{1+\delta^2}}\times\exp\left(\pm1.96\times\sqrt{\ln\left(1+\delta^2\right)}\right) \end{aligned} \tag{1A-2}$$

If the design site is a Finland site, the chance for the actual target value to be within the above nominal 95% CI (with b and δ calibrated by F-CLAY/7/216) should be close to 95%. If the design site is a general site, the chance for the actual target value to be within the above nominal 95% CI (with b and δ calibrated by CLAY/10/7490) should be close to 95%. The numbers of calibration data points (n) for some sand transformation models are quite limited (see Table 1A-2). For those transformation models, their nominal 95% CI may not be genuine.

Table 1A-1 Transformation models in the literature for some clay parameters.

Target parameter	Measured parameter(s)	Literature	Transformation model	Calibration database	Calibration results		
					n	Bias (b)	COV (δ)
s_u^{re}	LI	Locat and Demers (1988)	$s_u^{re}/P_a \approx 0.0144 \times LI^{-2.44}$	CLAY/10/7490	899	1.92	1.25
				F-CLAY/7/216	216	2.23	1.08
S_t	LI	Bjerrum (1954)	$S_t \approx 10^{0.8\times LI}$	CLAY/10/7490	1279	2.06	1.09
				F-CLAY/7/216	216	1.56	1.40
S_t	LI	Ching and Phoon (2012a)	$S_t \approx 20.726 \times LI^{1.910}$	CLAY/10/7490	1279	0.88	1.28
				F-CLAY/7/216	216	0.57	1.94
σ'_p	LI, S_t	Stas and Kulhawy (1984)	$\sigma'_p/P_a \approx 10^{1.11-1.62\times LI}$	CLAY/10/7490	249	2.94	1.90
				F-CLAY/7/216	67	7.54	1.13
σ'_p	LI, S_t	Ching and Phoon (2012a)	$\sigma'_p/P_a \approx 0.235 \times LI^{-1.319} \times S_t^{0.536}$	CLAY/10/7490	489	1.32	0.78
				F-CLAY/7/216	216	1.35	0.94
σ'_p	w_n, PL, LL	Kootahi and Mayne (2016)	If $5.512\log_{10}(\sigma'_v/P_a) - 0.061\times LL - 0.093\times PL + 6.219\times e_n > 1.123$ $\Rightarrow \sigma'_p/P_a \approx 1.62\times(\sigma'_v/P_a)^{0.89}(LL)^{0.12}(w_n)^{-0.14}$ Otherwise $\Rightarrow \sigma'_p/P_a \approx 7.94\times(\sigma'_v/P_a)^{0.71}(LL)^{0.53}(w_n)^{-0.71}$	CLAY/10/7490	1242	1.10	0.67
				F-CLAY/7/216	216	1.02	0.38
σ'_p	q_t	Kulhawy and Mayne (1990)	$\sigma'_p/P_a \approx 0.33\times(q_t - \sigma_v)/P_a$	CLAY/10/7490	690	0.97	0.39
			$\sigma'_p/P_a \approx 0.54\times(u_2 - u_0)/P_a$	CLAY/10/7490	690	1.18	0.75
σ'_p	q_t	Chen and Mayne (1996)	$\sigma'_p/P_a \approx 0.227\times[(q_t - \sigma_v)/P_a]^{1.200}$	CLAY/10/7490	690	0.99	0.42
			$\sigma'_p/P_a \approx 0.490\times[(q_t - u_2)/P_a]^{1.053}$	CLAY/10/7490	542	1.08	0.61
			$\sigma'_p/P_a \approx 1.274 + 0.761\times(u_2 - u_0)/P_a$	CLAY/10/7490	690	0.49	0.59
OCR	q_t	Kulhawy and Mayne (1990)	$OCR \approx 0.32\times(q_t - \sigma_v)/\sigma'_v$	CLAY/10/7490	690	1.00	0.39
OCR	q_t	Chen and Mayne (1996)	$OCR \approx 0.259\times[(q_t - \sigma_v)/\sigma'_v]^{1.107}$	CLAY/10/7490	690	1.01	0.42
			$OCR \approx 0.545\times[(q_t - u_2)/\sigma'_v]^{0.969}$	CLAY/10/7490	542	1.06	0.57

			$OCR \approx 1.026 \times B_q^{-1.077}$	CLAY/10/7490	779	1.28	0.86
s_u	PI	Mesri (1975)	$s_u/\sigma'_p \approx 0.22$	CLAY/10/7490	1155	1.04	0.55
				F-CLAY/7/216	216	1.08	0.28
s_u	OCR	Jamiolkowski et al. (1985)	$s_u/\sigma'_v \approx 0.23 \times OCR^{0.8}$	CLAY/10/7490	1402	1.11	0.53
				F-CLAY/7/216	216	1.15	0.29
s_u	OCR, S_t	Ching and Phoon (2012a)	$s_u/\sigma'_v \approx 0.229 \times OCR^{0.823} \times S_t^{0.121}$	CLAY/10/7490	395	0.84	0.34
				F-CLAY/7/216	216	0.84	0.32
s_u	q_t	Ching and Phoon (2012b)	$\left[(q_t - \sigma_v)/\sigma'_v\right]/(s_u/\sigma'_v) \approx 29.1 \times \exp(-0.513B_q)$	CLAY/10/7490	423	0.95	0.49
			$\left[(q_t - u_2)/\sigma'_v\right]/(s_u/\sigma'_v) \approx 34.6 \times \exp(-2.049B_q)$	CLAY/10/7490	428	1.11	0.57
			$\left[(u_2 - u_0)/\sigma'_v\right]/(s_u/\sigma'_v) \approx 21.5 \times B_q$	CLAY/10/7490	423	0.94	0.49

* All s_u are the "mobilized" s_u defined by Mesri and Huvaj (2007); e_n: natural void ratio.

Table 1A-2 Transformation models in the literature for some sand parameters.

Target parameter	Measured parameter(s)	Literature	Transformation model	Calibration database	Calibration results		
					n	Bias (b)	COV (δ)
D_r	$(N_1)_{60}$	Terzaghi and Peck (1967)	$D_r(\%) \approx 100 \times \sqrt{(N_1)_{60}/60}$	SAND/7/2794	198	1.05	0.231
D_r	N_{60}, OCR, C_u	Marcuson and Bieganousky (1977)	$D_r(\%) \approx 100 \times \left\{12.2 + 0.75\sqrt{\begin{matrix} 222 \times N_{60} + 2311 - 711 \times OCR \\ -779(\sigma'_{v0}/P_a) - 50 \times C_u^2 \end{matrix}}\right\}$	SAND/7/2794	132	1.00	0.211
D_r	$(N_1)_{60}$, OCR, D_{50}	Kulhawy and Mayne (1990)	$D_r(\%) \approx 100 \times \sqrt{\dfrac{(N_1)_{60}}{[60 + 25\log_{10}(D_{50})] \times OCR^{0.18}}}$	SAND/7/2794	199	1.01	0.205
D_r	q_{t1}	Jamiolkowski et al. (1985)	$D_r(\%) \approx 68 \times [\log_{10}(q_{t1}) - 1]$	SAND/7/2794	681	0.84	0.327
D_r	q_{t1}, OCR	Kulhawy and Mayne (1990)	$D_r(\%) \approx 100 \times \sqrt{\dfrac{q_{t1}}{305 \times Q_C \times OCR^{0.18}}}$	SAND/7/2794	840	0.93	0.339
ϕ'	D_r, ϕ'_{cv}	Bolton (1986)	$\phi' \approx \phi'_{cv} + 3 \times (D_r[10 - \ln(p'_f)] - 1)$	SAND/7/2794	391	1.03	0.052
ϕ'	D_r, ϕ'_{cv}	Salgado et al. (2000)	$\phi' \approx \phi'_{cv} + 3 \times (D_r[8.3 - \ln(p'_f)] - 0.69)$	SAND/7/2794	127	1.08	0.054
ϕ'	$(N_1)_{60}$	Hatanaka and Uchida (1996)	$\phi' \approx \sqrt{15.4 \cdot (N_1)_{60}} + 20$	SAND/7/2794	28	1.04	0.095
ϕ'	$(N_1)_{60}$	Hatanaka et al. (1998)	$\phi' \approx \begin{cases} \sqrt{15.4 \cdot (N_1)_{60}} + 20 & (N_1)_{60} \le 26 \\ 40 & (N_1)_{60} > 26 \end{cases}$	SAND/7/2794	58	1.07	0.090
ϕ'	$(N_1)_{60}$	Chen (2004)	$\phi' \approx 27.5 + 9.2 \times \log_{10}[(N_1)_{60}]$	SAND/7/2794	59	1.00	0.095
ϕ'	q_t	Robertson and Campanella (1983)	$\phi' \approx \tan^{-1}[0.1 + 0.38 \times \log_{10}(q_t/\sigma'_{v0})]$	SAND/7/2794	99	0.93	0.056
ϕ'	q_{t1}	Kulhawy and Mayne (1990)	$\phi' \approx 17.6 + 11 \times \log_{10}(q_{t1})$	SAND/7/2794	376	0.97	0.081

*ϕ_{cv}: critical-state friction angle (in degrees); p'_f is the mean effective stress at failure = $(\sigma'_{1f}+\sigma'_{2f}+\sigma'_{3f})/3$; Q_C = 1.09, 1.0, 0.91 for low, medium, high compressibility soils, respectively.

Discussion 1A – Sedimentary formations with heterogeneous 3D architecture

Celeste Jorge

Geological formations formed in a basin or a lacustrine environment, with endorheic regimen, are worldwide distributed, and are composed by continental detritic materials. These materials were deposited on a band, more or less wide, near the basin bank, giving rise to multiple alluvial fans. These geological formations are predominated by coarse materials (such as sandstones and conglomerates) and also by intermitted layers of calcareous materials (such as limestones and marls). The coarse materials are deposited during/immediately after torrential climatic periods, whereas the calcareous ones are deposited during calm periods. Furthermore, the basins were invaded successively by the ocean (advancements and retreatments of sea level over long geological periods). The episodes of advancements and retreatments of the sea level caused the deposition process highly variable and produce geological formations with significant three-dimensional (3D) heterogeneity.

The irregular structure and texture of these geological formations make it almost impossible to predict their spatial distribution. This feature is reflected in the great heterogeneity of the geotechnical characteristics. Thus, the use of probabilistic models for extrapolating parameters at unexplored locations may introduce a large error. Therefore, it is not advisable to use this type of extrapolation approach in such geological formations.

Reply to Discussion 1A

Jianye Ching

Celeste correctly pointed out that caution should be taken when parameters are extrapolated at unexplored locations. The highly spatially variable geological formations mentioned by Celeste serve as a very good example why such caution should be taken. I agree very much with Celeste. However, the development and implementation of a transformation model do not involve in spatial extrapolation.

To explain why the development of a transformation model does not involve in spatial extrapolation, let us consider the $(N_1)_{60}$-ϕ' model in Figure 1-1 as an example. Any data point in this figure is based on the test results of two sands in the literature. One sand is tested by the standard penetration test in situ to derive its $(N_1)_{60}$, whereas another sand sample is extracted using the ground freezing method and tested in laboratory to obtain its ϕ'. More importantly, the spatial locations of these two sands are typically very close, e.g., at the same depth and with few meters apart horizontally, to minimize the effect of spatial variation. As a result, these two sands are practically considered as the "same" sand, and the effect of spatial variation does not really enter into the data points in Figure 1-1.

The implementation of a transformation model does not involve in spatial extrapolation, either. For instance, suppose that an engineer would like to implement the transformation model (the blue dashed line in Figure 1-1) to estimate ϕ' for a sand at depth of 10 m with the knowledge of $(N_1)_{60}$ at the same depth. This process does not involve spatial extrapolation, either, because 10 m is the only depth of concern.

However, extrapolation can still happen when implementing a transformation model, but the extrapolation is not in space but in soil/rock type. For instance, the data points in Figure 1-1 are mostly from siliceous sands and there is no calcareous sand. If an engineer would like to implement the transformation model in Figure 1-1 to estimate ϕ' for a calcareous sand, extrapolation may happen.

The geological formations example mentioned by Celeste is indeed challenging in the aspect of spatial variability. However, transformation models are not for modeling and predicting spatial variability. The Discussion Group led by Dianqing Li (Incorporating spatial variability into geotechnical reliability based design) is more relevant to the subject of spatial variability.