EXCEL-based direct reliability analysis and its potential role to complement Eurocodes

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**Objectives & scope**
The similarities and differences between the design points of the first-order reliability method (FORM) and the Eurocode 7 (EC 7) design approach are studied. Eleven geotechnical examples of reliability analysis and reliability-based design (RBD) are discussed with respect to parametric correlations, sensitivity info from RBD, ultimate limit states and serviceability limit states, system reliability, spatially autocorrelated properties, and characteristic values and partial factors. Focus is on insights from RBD and how RBD can complement EC7 design approach in some situations, but limitations of RBD will also be mentioned.

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**Remarks**
Sessions 1, 2, 3, 4 and 5 by B K Low.
Session 6 by Yu Wang
Session 7 by KK Phoon, J Ching & Y Wang
Session 1: Introduction: practical Excel-based FORM reliability analysis.

- Excel-Solver automatic constrained optimization for implementing first-order reliability method (FORM)
- Intuitive perspective of expanding dispersion ellipsoid (or equivalent ellipsoid for non-Gaussian distributions) in the original space of random variables.
- Excel-Solver can in principle deal with more than hundred constraints and hundred changing cells
- The Excel VBA programming platform adds versatility, enabling FORM analysis involving implicit and iterative numerical model
- MATLAB and its optimization toolbox can also be used, but with bigger learning curve and less transparency to practitioners than Excel.
- Some Excel files for hands-on reliability analysis are available for download at http://alum.mit.edu/www/bklow

References:
Design points of FORM and EC7 compared

\[
\beta = \frac{R}{r}
\]

\textbf{Fig. 1.1} Illustration of the reliability index \( \beta \) in the plane when \( c' \) and \( \phi' \) are negatively correlated. This perspective is also valid for non-normal distributions, when viewed as "equivalent ellipsoids".

\textbf{tan}\( \phi' \) instead of \( \phi' \) can be used in the figure above without affecting the bulleted statements below.

\textbf{Fig. 1.2} Characteristic values, partial factors, and design approaches (DA) in Eurocode 7. (Low and Phoon, 2015)

- The design point in FORM reflects parametric uncertainties, sensitivities, and correlations, and is obtained without relying on partial factors.
- EC7 design point values, obtained by applying partial factors to characteristics values, are in general different from FORM design point values.
- The FORM reliability index \( \beta \) affords an estimation of the probability of failure. Design can aim at higher target \( \beta \) if consequence of failure is high.

General concepts of ultimate limit state design in Eurocode 7:

\( (\text{if } = \text{, then "design point"}) \)

- Diminished resistance (\( c_k / \gamma_c, \text{tan}\phi_k / \gamma_\phi \)) \geq \text{Amplified loadings}

- Characteristic values
- Partial factors
- Based on characteristic values and partial factors for loading parameters.

"Conservative", for example, 10 percentile for strength parameters, 90 percentile for loading parameters

The three sets of partial factors (on resistance, actions, and material properties) are not necessarily all applied at the same time.

In EC7, there are three possible design approaches:

- Design Approach 1 (DA1): (a) factoring actions only; (b) factoring materials only.
- Design Approach 2 (DA2): factoring actions and resistances (but not materials).
- Design Approach 3 (DA3): factoring structural actions only (geotechnical actions from the soil are unfactored) and materials.
Hypothetical statistical inputs using various non-Gaussian distributions to test Excel FORM and to discuss insights.

The mean value of spring stiffness $k_3$ at point 3 is 10, and that of rotational restraint $\lambda_1$ at point 1 is 500. Design point values from reliability analysis are indicated under the column labelled “$x_i^*$”.

Reliability analysis reveals that the design values of $\lambda_1$ and $k_3$ hardly deviate from their mean values. This means the buckling load $P_{\text{cr}}$ is insensitive to these two parameters at their mean values of 500 and 10.

In this case, the strut becomes a full sine curve, i.e. arching upwards on one side of point 3, and downwards on the other side, when $k_3 = 4$. Higher stiffness of $k_3$ serves no purpose once the full sine wave is formed.

This case may suggest characteristic values of $\lambda_1$ and $k_3$ equal to their mean values, and partial factors for both equal to 1.0, for the case in hand with inputs as shown.

For the case in hand, sensitivity of $k_3$ increases at lower values ($k_3 < 4$) when the strut has not gone into full sine wave yet.

Reliability-based design of $P$, $E$ and/or $I$ can be done for target $\beta$ index of 2.5 or 3.0.

---

**Fig. 1.3** Excel-Solver reliability analysis of a strut with complex supports. Performance function is implicit.
Limitations of partial factors back-calculated from reliability analysis

However, similar (but stiffer) spring and/or rotational restraint, if applied to the simple system on the left, or near the cantilever pile head of the laterally loaded pile on the right, would be important and sensitive parameters to the serviceability limit states of vertical displacement (left) and pile head deflection (right) and the ultimate limit states of spring rupture (left) and bending failure (right).

- Partial factors of spring stiffness $k$ back-calculated from reliability analysis are of different values within the same problem and across different problems.

- Hence direct FORM reliability analysis and reliability-based design (RBD) are preferred. Partial factors back-calculated from FORM will NOT be pursued in our discussion group, except when discussing the limitations of partial factors.

Fig. 1.4 Spring suspending a vertical load.

Fig. 1.5 A cantilever steel tubular pile in a pile group of a breasting dolphin.
Session 2: Reliability-based design of retaining walls

<table>
<thead>
<tr>
<th>H</th>
<th>γ'_{wall}</th>
<th>λ</th>
<th>α</th>
<th>γ_{soil}</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>24</td>
<td>10</td>
<td>90</td>
<td>18</td>
<td>0.4</td>
<td>1.83</td>
</tr>
</tbody>
</table>

\[
x^* = \mu + \sigma x \text{mean StDev } n = \frac{x^* - \mu}{\sigma}
\]

| tanϕ' | 0.492 | 0.7 | 0.07 | -2.977 |
| tanδ  | 0.262 | 0.36 | 0.036 | -2.725 |
| c_a   | 100   | 100 | 15  | 0.000 |

Correlation matrix

<table>
<thead>
<tr>
<th>tanϕ'</th>
<th>tanδ</th>
<th>c_a</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>0.8</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Rotating Sliding rotating mode sliding mode

- It is assumed that rotating mode is one of the ULS to be checked. Tanϕ', tanδ and c_a are normally distributed, with mean values (μ) and standard deviations (σ) as shown, and with correlation coefficient 0.8 between tanϕ' and tanδ.

- \( x^* \) are the design point values obtained in FORM reliability analysis. The column labelled \( n \) shows the values of \( (x^* - \mu) / \sigma \).

- For the statistical inputs shown, RBD obtains a design base width \( b \) = 1.83 m for a target \( \beta \) index of about 3.0, corresponding to a probability of rotation failure \( P_f = \Phi(-\beta) = 0.122\% \).

- For comparison, Monte Carlo simulation with 200,000 realizations using @RISK (www.Palisade.com) yields a \( P_f \) of 0.120%.

- The \( n \) value of 0.0 for \( c_a \) means the design value of \( c_a \) (under the \( x^* \) column) stays put at its mean value, because rotating mode is not affected by \( c_a \) at all. Reliability analysis reveals input sensitivities.

- Reliability analysis with respect to the ultimate limit states of sliding and bearing capacity failure can be done, and system reliability for multiple limit states can be evaluated readily (for example using the Low et al. 2011 system reliability procedure).

- Shown in the next page is EC7 design for the base width \( b \) with respect to the overturning ULS, via characteristic values and partial factors, starting from the same statistical inputs of mean values and standard deviations, but without considering correlations in EC7.

---

EC7 DA1b design of base width $b$ for rotation ULS

- Even though partial factors are specified, EC7 does not produce a unique design, but depends on how conservative the characteristic values are determined. This is not objectionable, for it allows flexibility in design to match the consequence of failure, in the same way that target reliability index can be higher or lower depending on the consequence of failure. Analogous situation exists for LRFD’s nominal values and load and resistance factors.

- For a design width $b$ obtained via EC7, the value of the corresponding reliability index $\beta$ is not unique, but depends on whether parametric correlations (if any) are modelled. To compare with the target $\beta$ of 3.0 in RBD (previous page), correlations should be modelled.

- EC7 DA1a requires characteristic values of resistance and actions, on which partial factors are applied. If characteristic values are based on percentiles, one needs to know the probability distributions of actions and resistance in order to estimate the upper tail (e.g. 70 percentile) characteristic value of actions and lower tail characteristic values of resistance (e.g. 30 percentile). Whether based on 5%/95% or 30%/70%, DA1a is satisfied; DA1b governs. Are there other ways of estimating characteristic values of actions and resistance for EC7 DA1a?

RBD of sheet pile total length H via FORM

- Free-earth support method. $K_a$ based on Coulomb formula; $K_p$ based on Kerisel-Absi.
- For the statistical inputs shown, RBD for a target $\beta = 3.0$ results in design $H (= 6.4 + z^* + d^*)$ of 12.31 m, and $P_f \approx \Phi(-\beta) = 0.13%$. For comparison, Monte Carlo simulation with 200,000 realizations gives $P_f = 0.14%$.
- For the statistical inputs shown, $\tan\phi'$ and z are sensitive inputs, based on $n$ values. The $n$ values of $\tan\delta$ and $\gamma$ are due largely to correlations with $\tan\phi'$, revealed if uncorrelated analysis is done.
- The design value of $\gamma$, 16.13, is lower than its mean value of 17, an apparent paradox which can be understood due to the logical positive correlation of $\gamma$ to $\gamma_{sat}$ and $\tan\phi'$ which both have design values below their respective mean values. If all six parameters are uncorrelated, the design value $\gamma^*$ will be bigger than mean $\gamma$.
- Soil on either side is assumed to be same source, hence one $\gamma_{sat}$ with action-resistance duality. Reliability analysis yields $\gamma_{sat}^* = 17.32$, which is less than the mean $\gamma_{sat}$ of 19.
- Mean embedment depth $d = 12.31 - 6.4 - 2.4 = 3.51$ m. Design embedment depth $d^* = 2.99$ m, i.e., “overdig” = 0.52 m, which is determined automatically as a by-product of RBD.
- Reliability analysis with respect to other ultimate limit states and system reliability analysis can be done.
- One can back-calculate partial factors for the six material and geometric inputs, and also load/action and resistance factors of LRFD and EC7 DA1a, based on presumed nominal/ characteristic values of loads/actions and resistance. Not pursued here due to limitations of such back-calculated partial factors.

References:
EC7 has an “unforeseen overdig” allowance for $z$, to account for the uncertainty of the dredge level. The design value of $z$ is obtained from $\mu_z + 0.5 \text{ m} = 2.4 + 0.5 = 2.9 \text{ m}$, where 0.5 m is the “overdig”.

Although EC7 partial factor of soil unit weight is specified to be 1.0, conservative characteristic values of $\gamma$ and $\gamma_{sat}$ still need to be estimated, and if same source, cannot increase unit weight on the active side while decrease unit weight on the passive side.

Based on 5/95 percentiles for characteristic values, $\gamma^* > \gamma_{sat}^*$, which violates reality.

With characteristic values at 30/70 percentiles and EC7 partial factors from DA1b, one obtains a design $H$ of 12.87 m, closer to the RBD design $H$ of 12.31 m for a target $\beta$ of 3.0. A less critical design $H$ of 12.64 m is obtained if one wrongly set characteristic value of $\gamma_{sat}$ at 19.52 instead of 18.48 kN/m$^3$.

FORM reliability analysis based on the $H$ from EC7 design will give different $\beta$ index depending on whether correlations are modeled (correlation matrix, previous page) or not.

**EC7 DA1b, 5/95 percentiles for $x_k$**

<table>
<thead>
<tr>
<th>$x_k$</th>
<th>Partial F</th>
<th>$x^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>18.48</td>
<td>1.00</td>
</tr>
<tr>
<td>$\gamma_{sat}$</td>
<td>17.36</td>
<td>1.00</td>
</tr>
<tr>
<td>$q_s$</td>
<td>13.29</td>
<td>1.30</td>
</tr>
<tr>
<td>$\tan\phi'$</td>
<td>0.688</td>
<td>1.25</td>
</tr>
<tr>
<td>$\tan\delta$</td>
<td>0.312</td>
<td>1.25</td>
</tr>
<tr>
<td>$Z$</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

**EC7 DA1b, 30/70 percentiles for $x_k$**

<table>
<thead>
<tr>
<th>$x_k$</th>
<th>Partial F</th>
<th>$x^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>17.47</td>
<td>1.00</td>
</tr>
<tr>
<td>$\gamma_{sat}$</td>
<td>18.48</td>
<td>1.00</td>
</tr>
<tr>
<td>$q_s$</td>
<td>11.05</td>
<td>1.30</td>
</tr>
<tr>
<td>$\tan\phi'$</td>
<td>0.754</td>
<td>1.25</td>
</tr>
<tr>
<td>$\tan\delta$</td>
<td>0.338</td>
<td>1.25</td>
</tr>
<tr>
<td>$Z$</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

$x_k$ = Characteristic value
$x^*$ = Design value
Computed EC7 design $H$ = 13.95 m
Note: Unrealistic that $\gamma^* > \gamma_{sat}^*$

Computed EC7 design $H$ = 12.87 m.
Computed EC7 design $H$ = 12.64 m if $\gamma_{sat,k}$ wrongly set at 70 percentile value (19.52) instead of the 30 percentile value (18.48).
How RBD can potentially complement EC7 design

Reliability-based design (RBD) can provide additional insights to EC7 design or LRFD design when the statistical information (mean values, standard deviations, correlations, probability distributions) of the key parameters affecting design are known and one or more of the following circumstances apply:

• When partial factors have yet to be proposed by EC7 to cover uncertainties of less common parameters, for example in situ stress coefficient K in underground excavations in rocks, dip direction and dip angles of rock discontinuity planes, and the smear effect of vertical drains installed in clay.

• When output has different sensitivity to an input parameter depending on the engineering problems (e.g. shear strength parameters in bearing capacity, retaining walls, slope stability).

• When input parameters are by their physical nature either positively or negatively correlated. One may note that EC7 does not specify different characteristic values and partial factors when some parameters are correlated. The same design is obtained in EC7 with or without modeling of correlations among parameters.

• When spatially autocorrelated soil properties need to be modeled.

• When there is a target reliability index or probability of failure for the design in hand. The target reliability index or probability of failure can be set explicitly higher or lower in RBD depending on the importance of the structure or consequence of failure.

• When uncertainty in unit weight of soil needs to be modelled.

One should not attach too precise a meaning to the results of RBD, because the output depends on input data, the mechanical model, the failure modes considered, etc. For example, in a RBD for target reliability index of 3.0, the resulting design is not to be regarded as having exactly a probability of failure equal to Φ(−β)=0.135%, but as a design aiming at sufficiently small probability of failure (<1%, say), in the same spirit that a EC7 design via characteristic values and partial factors and different ULS and SLS also aims at sufficiently safe design by implicit considerations of parametric uncertainties and sensitivities. In this regard, the statistical data and correlations are laid bare in RBD, and uncertainties, though imprecise, are explicitly accounted for.

Session 3: RBD of base width B of a retaining wall against bearing capacity failure mode

Similar to Fig. 3.1 of Low and Phoon (2015), except that $Q_h$ and $Q_v$ are uncorrelated, and the uncertainty in $\tan \phi'$ is modelled instead of $\phi'$, in line with EC7 which applies partial factor to $\tan \phi'$. The mean values follow those in Tomlinson (2001, page 94)'s Example 2.2.

For the statistical inputs shown, a base width $B$ of 4.55 m is required to achieve a target reliability index of $\beta = 3.0$ against bearing capacity failure.

The design value of $c'$, 15.03 kPa, is slightly above the $c'$ mean value of 15 kPa, due to negative correlation coefficient of -0.5 between $c'$ and $\tan \phi'$.

For the case in hand, the design is much more sensitive to $Q_h$ than to $Q_v$ with $n$ values 2.49 versus -0.71, and much more sensitive to $\tan \phi'$ than $c'$, with $n$ values -1.37 versus 0.11, where $n = (x^*-\mu_\text{N})/\sigma_\text{N}$, in which superscript N denotes equivalent normal mean and equivalent normal standard deviation of lognormal distributions.


(Continue next page)
RBD of base width $B$ of a retaining wall against bearing capacity failure mode (continued)

- The mean values of $c'$, $\tan \phi'$, $Q_h$ and $Q_v$ are 15 kPa, 0.47, 300 kN/m and 1100 kN/m, respectively, based on deterministic Example 2.2 in Tomlinson (2001, 7th ed., Foundation Design and Construction, p94), which computed an $F_s$ of 3.0 against general shear failure of the base of the wall when base width $B$ is 5 m.

- The RBD in the previous page assumes that the coefficients of variation of $c'$, $\tan \phi'$, $Q_h$ and $Q_v$ are 0.2, 0.1, 0.15 and 0.1, respectively. It is also assumed that $c'$ and $\tan \phi'$ are negatively correlated (as shown in the correlation matrix), but $Q_h$ and $Q_v$ are uncorrelated (as befitting the horizontal earth thrust and the applied vertical load).

- When $Q_h = 0$, the vertical load $Q_v$ is an unfavorable action without ambiguity. However, when $Q_h$ is acting and of comparable magnitude to $Q_v$, the latter possesses action-resistance duality, because load inclination and eccentricity decreases with increasing $Q_v$. RBD automatically takes this action-resistance (or unfavorable-favorable) duality into account in locating the design point. Interestingly, RBD reveals that the design value of $Q_v$ (1019.7 kN/m) is about 7.3% lower than its mean value of 1100 kN/m, thereby revealing the action-resistance duality of $Q_v$ when $Q_h$ is acting.

- Model uncertainty: The bearing capacity equation is approximate even for idealized conditions. Also, several expressions for $N_\gamma$ exist. The $N_\gamma$ used here is attributed to Vesic in Bowles (1996). The nine factors $s_j$, $d_j$ and $i_j$ account for the shape and depth effects of foundation and the inclination effect of the applied load. The formulas for these factors are based on Tables 4.5a and 4.5b of Bowles (1996), which may differ from those in EC7.

- RBD can be done for system reliability with multiple failure modes of ULS and/or SLS, as illustrated for a laterally loaded pile next.
RBD of a laterally loaded cantilever pile for ULS and SLS

- A steel tubular pile in a breasting dolphin. Soil-pile interaction was based on the nonlinear and strain-softening Matlock p-y curves.

- At mean input values of $P_H$ and undrained shear strength $c_u$, the pile deflection $y$ is 0.06 m at seabed, and 1 m at pile head.

- For reliability analysis, the $P_H$ was assumed to be normally distributed, with mean value 421 kN and a coefficient of variation of 25%. The mean $c_u$ trend is $\mu_{c_u} = 150 + 2z$, kPa, with a coefficient of variation of 30%. Spatial autocorrelation was modelled for the $c_u$ values at different depths below seabed.

- The $\beta$ index obtained was 1.514 with respect to yielding at the outer edge of the annular steel cross section. The sensitivities of $P_H$ and $c_u$ change with the cantilever length $e$. The different sensitivities from case to case is automatically revealed in reliability analysis and RBD, but will be difficult to consider in codes based on partial factors.

- A target $\beta$ of 3.0 can be achieved in RBD for both ULS (bending) and SLS (assuming $y_{\text{limit}} = 1.4$ m) using steel wall thickness $t = 32$mm and external diameter $d = 1.42$ m. Configurations along line $ac$ means $\beta = 3$ for bending mode, and greater than 3 for deflection mode; configurations along line $ab$ means $\beta = 3$ for deflection mode, and greater than 3 for bending mode.

(a) Laterally loaded cantilever pile in soil with Matlock p-y curves.

(b) depth-dependent Matlock p-y curves.

(c) RBD for both ULS and SLS

Fig. 3.2


Questions and possible discussions on bearing capacity ULS (pages 11 & 12) and ULS+SLS of the laterally loaded single pile with large cantilever length (page 13)

**Bearing capacity Ultimate Limit State**

How would partial factor design approaches (e.g. EC7 and LRFD) deal with a parameter that possesses action-resistance duality (i.e., unfavorable-favorable duality), such as the vertical load $Q_v$ in the presence of horizontal load $Q_h$?

**Laterally loaded pile, Ultimate Limit State and Serviceability Limit State**

- The case in hand (page 13) extends a deterministic example of Tomlinson (1994, Pile design and construction) into reliability analysis and RBD. The pile is one of a group of piles in a breasting dolphin, with 23 m embedment length below seabed and 26 m cantilever length in sea water. For serviceability limit state, it is assumed that the maximum permissible pile head deflection is 1.4 m. For both the bending ULS and the pile head deflection SLS, the design point in RBD shows decreasing sensitivity of $c_u$ with depth, i.e., decreasing $(c_u^* - \mu_{c_u})/\sigma_{c_u}$ with depth, where $c_u^*$ are the design undrained shear strength values at various depths obtained in RBD.

  How would partial factor design approaches determine the characteristic (or nominal) values of the undrained shear strength at different depths? Assuming uniform conservatism with depth in determining the characteristic $c_u$ values does not accord well with the different sensitivities of $c_u$ with depth as revealed by RBD, and may alter the behavior of the pile at ULS and SLS.

- If I understand correctly, for SLS design in EC7, all partial factors are equal to 1.0. So only conservative characteristic values need be determined for SLS design. For ULS design (e.g. bending of pile), having obtained the conservative $c_u$ characteristic values, should one apply the partial factor for $c_u$ uniformly across the entire embedded portion of the pile despite different sensitivities revealed in RBD?
**Session 4:** Soil slope reliability analysis, i.e. items II(e) and II(f) of Report.

Three examples are shown in the next three pages:

- Page 16: Reliability analysis of an underwater slope failure in San Francisco Bay Mud
- Page 17: Slope reliability analysis accounting for spatial autocorrelations
- Page 18: System reliability analysis involving multiple failure modes
Underwater excavated slope failure in San Francisco Bay Mud

A 75 m long section of the 600 m long trench failed during excavation in August 1970.

**Fig. 4.1** Underwater excavated slope in San Francisco Bay mud, and the average $s_u$ profile, described by $s_u = c_0 + b y$, and the ± 1 Std. Dev. lines.

Slope reliability analysis accounting for spatially autocorrelated undrained shear strength and soil unit weight

\[ c_u = \kappa \cdot (c_{u,\text{top}}(x) + \text{rate} \cdot \text{depth}) \]

Coefficient \( \kappa \) for anisotropy:
- \( \kappa = 0.4 \) if \( \alpha \leq -45^\circ \) (extension)
- \( \kappa = 1.0 \) if \( \alpha \geq 45^\circ \) (compression)
- \( \kappa \) linearly interpolated for \( -45^\circ \leq \theta \leq 45^\circ \)

\( \text{rate} = 2.85 \) kPa/m
\( \text{depth} = y_{\text{mid}, \text{top}} - y_{\text{mid}, \text{base}} \)

- Reliability analysis revealed that the slope is less safe when the unit weights near the toe are lower. This implication can be verified by deterministic runs using higher \( \gamma \) values near the toe, with resulting higher factors of safety.

- The design point (of 24 spatially correlated \( c_u \) values and 24 spatially correlated soil unit weight values) is located automatically in reliability analysis, and reflects parametric sensitivity from case to case in a way specified partial factors cannot.

- The computed reliability index and probability of failure depend on (i) the underlying analytical model, (ii) the number of parameters treated as random variables, and (iii) the statistical inputs including mean values, standard deviations, probability distributions, and correlation structure (including spatial variation and autocorrelation model where pertinent).

Fig. 4.2 Undrained shear strength model including anisotropy of a Norwegian slope.

System FORM reliability analysis of a soil slope with two equally likely failure modes.

The overlapping of the failure probability contents for modes 1, 3, 6 and 7, and also for modes 2, 4, 5 and 8 means that it is sufficiently accurate to calculate the bounds for the system failure probability by considering only the two stationary values of reliability index, namely $\beta_1$ and $\beta_2$.

Session 5: Rock slopes and tunnels in rock, i.e. items II(g), II(h) and II(i) of Report.

Three examples are shown in the next three pages:

- Page 20: Reliability analysis of Sau Mau Ping slope of Hong Kong;
- Page 21: Reliability analysis of 3-D tetrahedral wedge mechanism in rock slope;
- Page 22: Tunnel with roof wedge formed by discontinuities, and rock bolt reinforced tunnel.
Reliability-based design of Sau Mau Ping slope of Hong Kong

**Units**: meter, tonne, tonne/m², tonne/m³.

- The statistical inputs of the 5 random variables follow those in Hoek (2007). For zero reinforcing force $T$ and uncorrelated parameters, the FORM reliability index is $\beta = 1.556$, and $P_f \approx 1 - \Phi(\beta) = 6\%$, in good agreement with the Monte Carlo $P_f$ of 6.4% in Hoek (2007).
- With negative correlations between $c$ and $\phi$ and between $z$ and $zw/z$, as shown in the correlation matrix $R$, a reinforcing force $T$ of 257 tons (per m length of slope) inclined at $\theta = 55^\circ$ is needed to achieve a target reliability index $\beta$ of 3.0. The most sensitive parameters for the case in hand are the ratio $zw/z$ and the coefficient of horizontal earthquake acceleration $\alpha$.
- The tension crack depth $z$ and the extent to which it is filled with water ($zw/z$) are negatively correlated. This means that shallower crack depths tend to be water-filled more readily (i.e., $zw/z$ ratio will be higher) than deeper crack depths, consistent with the scenario suggested in Hoek (2007) that the water which would fill the tension crack in this Hong Kong slope would come from direct surface run-off during heavy rains.
- For illustrative purposes, a negative correlation coefficient of -0.5 is assumed between $z$ and $zw/z$. Even though there are no data to quantify this correlation between $z$ and $zw/z$, it is still useful to explore possible correlations to get a feel for its influence on the reliability index. This is a sensible approach commonly applied in engineering practice for important but not well characterized parameters.

![Diagram of Sau Mau Ping slope](image)

**Fig. 5.1**


Reliability analysis of 3-D wedge in rock slope with uncertain discontinuity orientations

Tetrahedral wedge

Triangle BDE is horizontal.
Lines TS and XR, are horizontal.

\( \beta_1 \) and \( \beta_2 \) are related to strike directions; \( \delta_1 \) and \( \delta_2 \) are dip angles.
\( \alpha \) is the slope face inclination, \( \Omega \) the upper ground inclination.
Slope face dip direction coincides with upper ground dip direction.

\[ H \text{ and } h \] are related by equation, either can be input.

The uncertainties of discontinuity orientations (\( \beta_1, \delta_1, \beta_2, \delta_2 \)), shear strength of joints (\( \tan \phi \) and \( c/\gamma h \)), and water pressure in joints (dimensionless parameter \( G_w \)) are modelled by the versatile beta general distributions which can assume non-symmetrical bounded pdf. Here the beta-distribution parameters used are \((4, 4, \text{mean } - 3\text{StDev}, \text{mean } + 3\text{StDev})\). If desired, other values of lower and upper bounds and mode \( \neq \text{mean} \) can be modelled.

- The reliability analysis here assumes the means and standard deviations of \( \tan \phi, c/\gamma h \) and \( G_w \) on joint plane 1 are identical to those on joint plane 2. These assumptions are for simplicity, not compulsory.

- Reliability analysis yielded \( \beta = 1.924 \) against sliding on both planes, \( \beta = 1.389 \) against sliding on plane 1, and \( \beta > 5 \) for other modes.

- Although the governing failure mode at mean values is sliding on both planes, the reliability index \( \beta \) against sliding on plane 1 is—in the presence of uncertainty in discontinuity orientations (\( \beta_1, \delta_1, \beta_2, \delta_2 \)—more critical than that against sliding on both planes. This information would not be revealed in a deterministic analysis, or in a reliability analysis that considers only one failure mode.

Reliability analysis of tunnels in rocks

(a) Roof wedge in tunnel

(b) Tunnel with rock bolts

(c) A tale of two factors of safety, and reconciliation via reliability analysis

- One can design the rockbolt spacings $S_2$ and $S_{\theta}$ and the rockbolt length so as to achieve a target reliability index, for example $\beta = 3.0$, corresponding to a probability of failure of about 0.1%. Examples of reliability-based designs are given in the paper.

- In its current version, EC7 covers little on the characteristic values and partial factors of rock engineering parameters like orientations of discontinuities, in situ stresses, and properties of joints and rock material. RBD is a more flexible approach in dealing with case-specific uncertainties of input values and can potentially complement EC7 (and LRFD)

Session 6: Excel-based Subset Simulation

- Subset simulation (Au and Beck 2001) is an advanced Monte Carlo Simulation (MCS) that aims to improve MCS’s computational efficiency, particularly at probability tails, while maintaining its robustness.

- Subset Simulation stems from the idea that a small failure probability can be expressed as a product of larger conditional failure probabilities for some intermediate failure events, thereby converting a rare event (small probability levels) simulation problem into a sequence of more frequent ones. Subset Simulation is performed level by level. The first level is direct MCS, and the subsequent levels utilize Markov chain Monte Carlo to generate conditional samples of interest. Details on Subset simulation are referred to Au and Beck (2001) and Au and Wang (2014).

- An Excel VBA Add-in called “Uncertainty Propagation using Subset Simulation” (UPSS) has been developed and can be obtain from https://sites.google.com/site/upssvba (Au et al. 2010, Au and Wang 2014).

- UPSS divides the reliability analysis or design into three separate processes: (1) deterministic modeling, (2) uncertainty modeling, and (3) uncertainty propagation by Subset simulation. The deterministic modeling is deliberately decoupled from uncertainty modeling and propagation. This allows three separate processes mentioned above to proceed in a parallel fashion. The uncertainty modeling and propagation are performed in a non-intrusive manner, and the robustness of MCS is well maintained. This removes the mathematical hurdles for engineering practitioners when performing reliability analyses or designs.


Subset Simulation Application Example

Excel-based Subset simulation has been used in reliability analysis of slope stability (Wang et al. 2010&2011, Wang and Cao 2015) and reliability-based design of foundation (Wang and Cao 2013&2015). A slope stability example is illustrated in this page. Details of the example are referred to Wang et al. (2011).

- Subset simulation significantly improves computational efficiency and resolution, particularly at small probability levels.
- When the spatial variability of soil properties is considered, the critical slip surface varies spatially. Using only one given critical slip surface significantly underestimates failure probability, and it is unconservative. Thus, when the soil property spatial variability is considered, the spatial variability of the critical slip surface should be properly accounted for.
Potential merits of MCS/Subset Simulation in EC7

EC7 adopts design formats similar to the traditional allowable stress design (ASD) methods. The factor of safety in ASD methods is replaced by a combination of partial factors in EC7, which are provided after some code calibration processes. Because design engineers are not involved in the calibration processes, many assumptions and simplifications adopted in the calibration processes are frequently unknown to the design engineers. This situation can lead to potential misuse of the partial factors that are only valid for the assumptions and simplifications adopted in the calibration processes. Design engineers may feel uncomfortable to accept these “black box” calibration processes blindly. In addition, design engineers have little flexibility in changing any of these assumptions/simplifications or making their own judgment because recalibrations are necessary when any assumption or simplification is changed.

MCS/Subset Simulation has potential merits in the aforementioned aspects. Because MCS/Subset Simulation can be treated as repeated computer (Excel) executions of the traditional ASD calculations, good geotechnical sense and sound engineering judgment that have been accumulated over many years of ASD practice are well maintained during the development of the deterministic (ASD) model for MCS/Subset simulation. The MCS/Subset simulation – based design can be conceptualized as a systematic sensitivity study which is common in geotechnical practice and familiar to design engineers. MCS/Subset simulation is particularly beneficial in the following situations (Wang et al. 2016):

- When the target failure probability needed in the design is different from the target failure probability pre-specified in EC7.
- When the exact value of failure probability is needed in engineering applications, such as quantitative risk assessment and risk based decision making.
- When the load and resistance are correlated. For example, the load and resistance for earth retaining structures and slopes are usually originated from the same sources (e.g., effective stress of soil) and correlated with each other. It is therefore difficult to decide whether the effective stress of soil or earth pressure should be regarded as a load or resistance.
- When dealing with geometric uncertainties, such as orientation of joints in rock engineering. The geometric uncertainties cannot be easily considered by conventional partial factors.

Session 7

Probabilistic models for geotechnical data

KK Phoon, J Ching & Y Wang

Introduction

This section provides simple guidelines on how to fit geotechnical data (soil parameters and model factors) to practical probabilistic models.

Geotechnical parameters

1. Single random variable – normal
2. Single random variable – Johnson (lognormal is a special case)
3. Random vector – normal
4. Random vector – Johnson
5. EXCEL Add-in for Bayesian Equivalent Sample Toolkit (BEST)
6. Statistical guidelines

Calculation models

7. Model factors

Geotechnical data can come in the form of 1 column of numbers in an EXCEL spreadsheet, say cone tip resistance with depth. Model factors for one response (defined as the ratio of measured to calculated response) also appear as one column of numbers. The number of rows occupied by the numbers is called the “sample size”. We call this column “univariate data”.

Univariate data can be fitted by a single random variable. This random variable is fully defined once we identify the distribution type (e.g., normal, lognormal) and we evaluate the parameters of the distribution (e.g. mean, standard deviation).

Geotechnical data can come in the form of several columns of numbers in an EXCEL spreadsheet, say cone tip resistance, sleeve friction, and pore pressure with depth. Each row refers to several measurements taken at the same location and at the same depth. Model factors for multiple responses (say deflection and bending moment of a cantilever wall) will appear as two column of numbers. We call this group of columns “multivariate data”.

Multivariate data can be fitted by a random vector. This random vector is fully defined once we identify the joint distribution type (the most common is multivariate normal) and we evaluate the parameters of the distribution (e.g. mean, standard deviation, and correlation matrix). For an EXCEL spreadsheet containing the cone tip resistance, sleeve friction, and pore pressure, the required joint distribution is a 3-dimensional generalization of the more widely known probability density functions in basic texts. This generalization is simple in concept but difficult to actualize in practice, because the columns are related to each other in interesting ways (called a “dependency” structure).

More advanced aspects are not covered to keep this guideline accessible to the general reader. Nonetheless, one should be mindful of the following:

1. Are my data “random”?

Data must be “homogeneous” or “stationary”. In simple terms, this means that the data must appear “random” without exhibiting an obvious trend with depth or other auxiliary variables.
A practical approach is to remove the trend when it exists.

There are more subtle kind of non-randomness beyond presence of an average trend. There are formal hypothesis tests to address this.

2. Are my data normal or lognormal?

A histogram is a rather intuitive way to present the range and frequency characteristics of the data. However, lack of fit of a theoretical distribution to a histogram does not imply that the distribution is inadequate, particularly for small sample size.

A practical approach is to compare the empirical and theoretical cumulative distribution function, rather than the histogram and probability density function. There are more formal “goodness of fit” tests to address this aspect in a robust manner.

3. Do I have sufficient data?

The mean, standard deviation, correlation matrix, or other parameters can be estimated from a finite sample size using a variety of methods. Each set of data produces a single value for the mean, standard deviation, and correlation matrix. These point estimates produced by data (called statistics) are affected by “statistical uncertainty”. Statistical uncertainty can be presented as a 95% confidence interval (95% CI). The classical interpretation is that there is a 95% chance of capturing the true value of the mean (or other statistics) within this interval. A small sample size produces a wide 95% CI and in the extreme, the statistics are not meaningful.

The key idea is that statistical uncertainty allows the engineer to answer the practical question “do I have sufficient data?” in a concrete way. This question is critical in geotechnical engineering, because our sample sizes are typically small.

There are general statistical methods to address this, but more work is needed to develop practical tools or softwares for engineers.

4. Can I combine my site investigation data to reduce uncertainties?

The reader is referred to the discussion topic on “Transformation uncertainties & multivariate soil data” for more details. The central idea can be explained using Figure 1 below:
In the absence of site-specific data but in the presence of data from comparable sites, the engineer may assess the effective stress friction angle ($\phi'$) to fall between $28^\circ$ and $51^\circ$ based on the scatter of “×” markers. However, if site-specific SPT N-values are available and they fall in the vicinity of 25 blows, it is possible to reduce the uncertainty in $\phi'$ because the “Θ” markers fall within a more restrictive range of $36^\circ$ and $46^\circ$.

How to perform “updating” in the presence of several tests? This task can be performed systematically and consistently within a powerful Bayesian framework. More work is needed to develop practical tools or softwares for engineers, but calculations are a lot simpler if the multivariate normal distribution is applicable.

The guidelines below focus on “how to use” using EXCEL, rather than the theory. The user may like to refer to Ching & Phoon (2015) for a more complete treatment that includes the advanced aspects highlighted above.

**Single random variable – normal**

There are good reasons to consider the normal distribution as a default distribution (Phoon 2006). It is possible to fit data to any distribution, but generalization to higher dimensions (which is necessary to cover multiple test measurements in a site investigation program) is more difficult.

The normal distribution is completely defined by the mean and standard deviation, which can be estimated by applying the EXCEL “average” and “stdev.s” functions on one column of data.

Normal data can be simulated using the mean and standard deviation as inputs (refer to Data Analysis > Random Number Generation under the Data tab in EXCEL).

In principle, the normal distribution is not suitable for geotechnical parameters taking values greater than zero (positive-valued parameters such as strength parameters).

Nonetheless, it can be used for positive-valued parameters as an approximate distribution when the COV is “small”.

---

**Figure 1. Relationship between effective stress friction angle and SPT blowcount.**

In the absence of site-specific data but in the presence of data from comparable sites, the engineer may assess the effective stress friction angle ($\phi'$) to fall between $28^\circ$ and $51^\circ$ based on the scatter of “×” markers. However, if site-specific SPT N-values are available and they fall in the vicinity of 25 blows, it is possible to reduce the uncertainty in $\phi'$ because the “Θ” markers fall within a more restrictive range of $36^\circ$ and $46^\circ$.
One concrete way of checking suitability of normal distribution is to check the design point produced by the First Order Reliability Method (FORM). If the design point is negative for the positive-valued parameter, then the normal distribution is not suitable.

Another rough check for strength parameters is: \( (1 – \text{target reliability index} \times \text{COV}) > 0 \). For example:

- target reliability index = 3 and COV = 0.1 ⇒ ok
- target reliability index = 3 and COV = 0.5 ⇒ not ok

If a normal distribution is not suitable, one should try to select from the Johnson system, in which the ubiquitous lognormal distribution is one member of this system.

**Single random variable – Johnson (lognormal is a special case)**

Let \( Y \) be a soil parameter that cannot be fitted to a normal distribution. The Johnson system allow \( Y \) (which is non-normal) to be calculated from a standard normal random variable \( X \) analytically:

\[
\frac{X - b_X}{a_X} = \kappa\left(\frac{Y - b_Y}{a_Y}\right) = \kappa(Y)
\]

It suffices to note that the Johnson system can generate distributions with a wide range of shapes by choosing the function \( \kappa(.) \), followed by calibration of 4 parameters, \((a_X, b_X, a_Y, b_Y)\). There are only 3 possible functions for \( \kappa(.) \) and they produce the SU, SB, and SL distributions. The SL distribution is produced by \( \kappa(.) = \ln(.) \), which is the well-known lognormal distribution. There is a simple procedure to do this (Section 1.5.3, Ching & Phoon 2015). Examples are shown in Figure 2 below.

(a) SU distribution (Baseline is with \( a_X = 1, b_X = 0, a_Y = 1, b_Y = 0 \)
Figure 2. Examples of SU, SB, and SL probability distributions from the Johnson system.
Random vector – normal

For concreteness, assume that you have an EXCEL spreadsheet containing 3 columns. Column A contains data for the cone tip resistance, column B contains data for the sleeve friction, and column C contains data for pore pressure. We further assume that these 3 measurements were taken at 100 points in the depth direction. Therefore, the data is contained within the block of cells from A1 to C100.

If the data are normally distributed, we can build a 3-dimensional normal random vector which consists of the following collection of random variables \((Z_1, Z_2, Z_3)\). The random variable \(Z_1\) is for cone tip resistance and so forth for \(Z_2\) and \(Z_3\).

It is easy to calculate the mean \((\mu_1)\) and standard deviation \((\sigma_1)\) for \(Z_1\) using the EXCEL “average” and “stdev.s” functions on each column of data. The means and standard deviations for \(Z_2\) and \(Z_3\) are obtained in the same way.

The key difference between a random variable and a random vector is a “correlation matrix”, containing the correlation between cone tip resistance and sleeve friction \((\delta_{12})\), the correlation between cone tip resistance and pore pressure \((\delta_{13})\), and the correlation between sleeve friction and pore pressure \((\delta_{23})\). You can get this correlation matrix directly from the data block A1:C100 using “Data Analysis > Correlation” under the Data tab in EXCEL.

<table>
<thead>
<tr>
<th></th>
<th>(Z_1)</th>
<th>(Z_2)</th>
<th>(Z_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z_1)</td>
<td>1</td>
<td>(\delta_{12})</td>
<td>(\delta_{13})</td>
</tr>
<tr>
<td>(Z_2)</td>
<td>(\delta_{12})</td>
<td>1</td>
<td>(\delta_{23})</td>
</tr>
<tr>
<td>(Z_3)</td>
<td>(\delta_{13})</td>
<td>(\delta_{23})</td>
<td>1</td>
</tr>
</tbody>
</table>

Once you obtained this correlation matrix, you can refer to the following sections in Ching & Phoon (2015) for simulation (Section 1.4.4) and Bayesian updating (Section 1.4.5).

Random vector – Johnson

Geotechnical data are multivariate and non-normal, including the example containing 3 columns for the cone tip resistance, sleeve friction, and pore pressure. The standard approach to handle this is to transform each column of non-normal data into a column of normal data using the Johnson system as described above.

Once you obtain 3 columns of normal data, you can use the procedure described in the preceding section for simulation and Bayesian updating. Computational details are given in Section 1.6 and 1.7 (Ching & Phoon 2015) and applications to actual soil databases are given in Ching et al. (2016).

EXCEL Add-in for Bayesian Equivalent Sample Toolkit (BEST)

Although an analytical probability distribution function (e.g., normal or Johnson) is frequently used to fit measurement data and represent a random variable, it is also possible to empirically represent a random variable using a large number of numerical samples simulated from the random variable. This is particularly beneficial when the number of measurement data (i.e., sample size) is too small to directly and properly fit the probability density function (e.g., the 95% CI is too large and the statistics are not meaningful). To deal with the issue of small sample size, Bayesian methods may be used to integrate limited measurement data in a specific site with prior knowledge (e.g., engineering experience and judgment, existing data from similar project sites) to provide updated knowledge on
the soil parameter of interest (e.g., Wang et al 2016a). Because the updated knowledge might be complicated and difficult to express explicitly or analytically, Markov chain Monte Carlo (MCMC) simulation has been used to transform the updated knowledge into a large number of simulated samples of the soil parameter of interest, which collectively represent the soil parameter as a random variable (Wang and Cao 2013). An EXCEL add-in, called Bayesian Equivalent Sample Toolkit (BEST), has been developed for implementing the Bayesian method and MCMC simulation in a spreadsheet platform (Wang et al. 2016b). The BEST Add-in can be obtained without charge from https://sites.google.com/site/yuwangcityu/best/1. Engineering practitioners only need to provide input to the BEST Add-in, such as site-specific measurement data (e.g., several SPT N values) and typical ranges of soil parameters of interest (e.g., effective friction angle of soil) as prior knowledge. Then, the BEST Add-in may be executed to generate a large number of numerical samples of the soil parameters. Subsequently, conventional statistical analysis can be performed on these simulated samples using EXCEL’s built-in functions (e.g., “average” and “stdev.s”). The BEST Add-in can also be used for estimating soil parameters (e.g., undrained shear strength of clay) from “multivariate data” (e.g., SPT, CPT, and Liquidity index data) in a sequential manner using a Bayesian sequential updating method (Cao et al. 2016).

**Statistical guidelines**

Extensive statistics have been compiled in the literature. These statistics are summarized by Phoon et al. (2016) and Ching et al. (2016).

The coefficient of variation (COV) is defined as the ratio of the standard deviation to the mean. Guidelines for COV for soil and rock parameters are given in Phoon et al. (2016a). It is important to note that COV of a soil or rock parameter can small or large, depending on the site condition, the measurement method, and the transformation model. Resistance factors should be calibrated using the three-tier COV classification scheme shown in Table 1 to provide some room for the engineer to select the resistance factor that suits a particular site and other localized aspects of geotechnical practice (e.g. property estimation procedure) (Phoon et al. 2016b). A single resistance/partial factor ignores site-specific issues and it shares the same issues as the factor of safety approach where the nominal resistance has to be adjusted to handle site-specific considerations in the presence of a relatively constant factor of safety.

<table>
<thead>
<tr>
<th>Geotechnical parameter</th>
<th>Property variability</th>
<th>COV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undrained shear strength</td>
<td>Low&lt;sup&gt;a&lt;/sup&gt;</td>
<td>10 - 30</td>
</tr>
<tr>
<td></td>
<td>Medium&lt;sup&gt;b&lt;/sup&gt;</td>
<td>30 - 50</td>
</tr>
<tr>
<td></td>
<td>High&lt;sup&gt;c&lt;/sup&gt;</td>
<td>50 - 70</td>
</tr>
<tr>
<td>Effective stress friction angle</td>
<td>Low&lt;sup&gt;a&lt;/sup&gt;</td>
<td>5 - 10</td>
</tr>
<tr>
<td></td>
<td>Medium&lt;sup&gt;b&lt;/sup&gt;</td>
<td>10 - 15</td>
</tr>
<tr>
<td></td>
<td>High&lt;sup&gt;c&lt;/sup&gt;</td>
<td>15 - 20</td>
</tr>
<tr>
<td>Horizontal stress coefficient</td>
<td>Low&lt;sup&gt;a&lt;/sup&gt;</td>
<td>30 - 50</td>
</tr>
<tr>
<td></td>
<td>Medium&lt;sup&gt;b&lt;/sup&gt;</td>
<td>50 - 70</td>
</tr>
<tr>
<td></td>
<td>High&lt;sup&gt;c&lt;/sup&gt;</td>
<td>70 - 90</td>
</tr>
</tbody>
</table>

<sup>a</sup> - typical of good quality direct lab or field measurements  
<sup>b</sup> - typical of indirect correlations with good field data, except for the standard penetration test (SPT)  
<sup>c</sup> - typical of indirect correlations with SPT field data and with strictly empirical correlations
**Model factors**

The model factor for the capacity of a foundation is commonly defined as the ratio of the measured (or interpreted) capacity \(Q_m\) to the calculated capacity \(Q_c\), i.e. \(M = Q_m / Q_c\). The value \(M = 1\) implies that calculated capacity matches the measured capacity, which is unlikely for all design scenarios. Intuition would lead us to think that \(M\) takes different values depending on the design scenario. This intuitive observation is supported by a large number of model factor studies (Dithinde et al. 2016). Hence, it is reasonable to represent \(M\) as a random variable. It is straightforward to apply this simple definition to other responses beyond foundation capacity. For some simplified calculation models, \(M\) can depend on input parameters (i.e., \(M\) is not random) and additional efforts are required to remove this dependency (Zhang et al. 2015).

A comprehensive summary of model factor statistics is presented by Dithinde et al. (2016). Multivariate model factors are not available at present.

**Key references**

**Conclusions**

**Merits and limitations of FORM reliability analysis and RBD** as illustrated in the 11 examples in the first five sessions of “EXCEL-based direct reliability analysis and its potential role to complement Eurocodes”

Reliability-based design (RBD) can provide additional insights to EC7 design or LRFD design when the statistical information (mean values, standard deviations, correlations, probability distributions) of the key parameters affecting design can be estimated and one or more of the following circumstances apply:

- When partial factors have yet to be proposed by EC7 to cover uncertainties of less common parameters, for example in situ stress coefficient $K$ in underground excavations in rocks, dip directions and dip angles of rock discontinuity planes, and other miscellaneous parameters in soil or rock engineering.
- When output has different sensitivity to an input parameter depending on the engineering problems, e.g. sensitivity of shear strength parameters in bearing capacity, retaining walls, slope stability etc. may be different.
- When there is action-resistance duality in a parameter, for example the vertical load in bearing capacity when horizontal load is also acting (Session 3).
- When input parameters are by their physical nature either positively or negatively correlated.
- When spatially autocorrelated soil properties need to be modelled.
- When there is a target reliability index or probability of failure for the design in hand. The target reliability index or probability of failure can be set explicitly higher or lower in RBD depending on the importance of the structure or consequence of failure.
- When uncertainty in unit weight of soil needs to be modelled.

The design point in RBD is the most probable failure combination of parametric values for a target reliability index, and reflects case-specific uncertainties, sensitivities, probability distributions and parametric correlations in a way that prescribed partial factors and “conservative” characteristic values cannot.

**LIMITATIONS:** Statistical inputs are approximate and often involve judgment, due to insufficient data. Also, may have overlooked some factors (e.g. human factors). Hence the probability of failure based on $P_f \approx 1 - \Phi(\beta)$ is not exact. (Despite imprecise statistical inputs, RBD may still achieve its aim of “sufficiently safe designs”).
References for Sessions 1 to 5:


Also, 8 references are listed in pages 23-25 (Session 6) by Y Wang,

and 12 references are listed in page 33 (Session 7, pages 26-33) by KK Phoon, J Ching & Y Wang