PART2 TC205-TC304 DRAFT REPORT

Discussion on Imprecise Probabilistic and Interval Approaches Applied to the Eurocode 7 Partial Factor Design

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ABSTRACT

The basis on engineering design is the realistic modelling of processes. However, imprecise information entangles the specification of a unique probabilistic model. From first developments, imprecise probabilistic approaches are attracting increasing attention to address this shortcoming. A pathway for such investigation is demonstrated on the analysis of a strip spread foundation designed by the Eurocode 7 methodology. The shear strength parameters of the foundation soil are implemented as intervals based on which the characteristic values for design are derived. On this case study the shear strength parameter friction angle of the foundation soil is further separately implemented as interval in the format of a conditional analysis. A limit state imprecise probabilistic grid-based or fuzzy-based approach applied to the Eurocode 7 partial factor design for bearing capacity safety assessment is then pursued in the format of a sensitivity analysis for a group of model cases, considered for demonstration only normal variabilities. At last, the state of play on the assessment of robustness is extended to the Eurocode 7 partial factor design. Imprecise approaches to robust design are then discussed on the calculation of resistance factors capable to maintain a more uniform reliability level over a range of design parameters.

INTRODUCTION

EN 1997, adopted as Eurocode 7, is intended to be applied to the geotechnical aspects of the design of civil engineering works. The limit state design concept adopted by Eurocode 7 is used in conjunction with a partial factor methodology. The selection of appropriate partial factors is important to ensure the reliability of geotechnical design to Eurocode 7, as design values are determined by applying partial factors to characteristic values. Orr (2013) further explains that to achieve the required target reliability, Eurocode 7 does not provide any variation in the partial factors but rather requires that greater attention is given to other accompanying measures related to design supervision and inspection differentiation by a system of failure control.

Considered that the performance of a partial factor format is measured by the ability to produce a design achieving a target reliability within acceptable error in the considered domain or subdomain, Phoon and Ching (2013) argue about the shortcomings of a constant partial factor format and explain that the capability to maintain a uniform reliability level is primarily related to the variability of scenarios and the number of available partial factors. Thus, the state of play on the assessment of geotechnical robustness has been recently extended to partial factor design. A modern concept of robustness expresses the degree of independence among any changes in the whole set of parameters and the fluctuation on the response considered a global specification on a minimum variance with respect to input variations. In this way, new approaches have been advanced for the calculation of factors capable to maintain a more uniform reliability level over a range of design parameters and regional studies are to an increasing extent providing revisions of factors taking into account the local uncertainties involved and the reliability theory.

This discussion on imprecise probabilistic approaches applied to the Eurocode 7 partial factor design is based on the analysis of a strip spread foundation for bearing capacity safety assessment, considered for demonstration only normal variabilities. The underlying idea in the imprecise probability theory consists in modelling an imprecise probability distribution by a set of candidate probability distributions which are derived from the available data, so that a probability bounding approach is applied to specify lower and upper bounds. Thereby, the indecision interval reflects the imprecision of the model derived from the set of competing intervals. According to Marques *et al.* (2015a), the replacement of one exact probability value by an indecision interval with two different exact endpoints introduces new arguments about the consistency of the indecision interval. In fact, broader approaches may include a mixed set of probabilistic and nonprobabilistic interval models wherein different bounding measures may be applied in order to find the limit state lower and upper bounds, see Marques *et al.* (2015b).

IMPRECISE PROBABILISTIC GRID AND FUZZY ANALYSIS

Uncertainty is recognised now as a central feature of geotechnical engineering and among a number of strategies, quantifying uncertainty is the great purpose of reliability approaches. Thus, the geotechnical engineer must increasingly be able to deal with reliability in order to provide new insights in professional practice. This transition has been notedly characterised by a natural resistance in the application of probabilistic approaches. In fact, probability fails to incorporate factors that are ignored in the analysis, but it is indeed very useful to compare several alternative designs and moreover, uncertainty contributions of different components are revealed in a sensitivity analysis. However, in order to develop appropriate input, the nature of uncertainty and probability must be understood. For instance, properties estimated from small samples may be seriously in error, whether they are used deterministically or probabilistically.

The basis for a reliable engineering analysis is the appropriate treatment of information in accordance with the underlying real world. However, information is not certain but rather imprecise and a sensitivity analysis may be pursued through probabilistic grid and fuzzy approaches on multiple intervals. Thereby, the imprecise probability theory emerges as a base for decision-making when providing the investigation of the most plausible models. The key feature concerns on the identification of probability bounds for scenarios of interest which reflect the uncertainty as the range between the limits. The imprecise probabilistic analysis is then a supplementary element which enriches the variety of models to be combined with the traditional overview in improved adaptability. These models may include interval and fuzzy probabilities in association to a probability box structure constructed from search amid the competing models, see Marques *et al.* (2015a) and sketch on Figure 1.

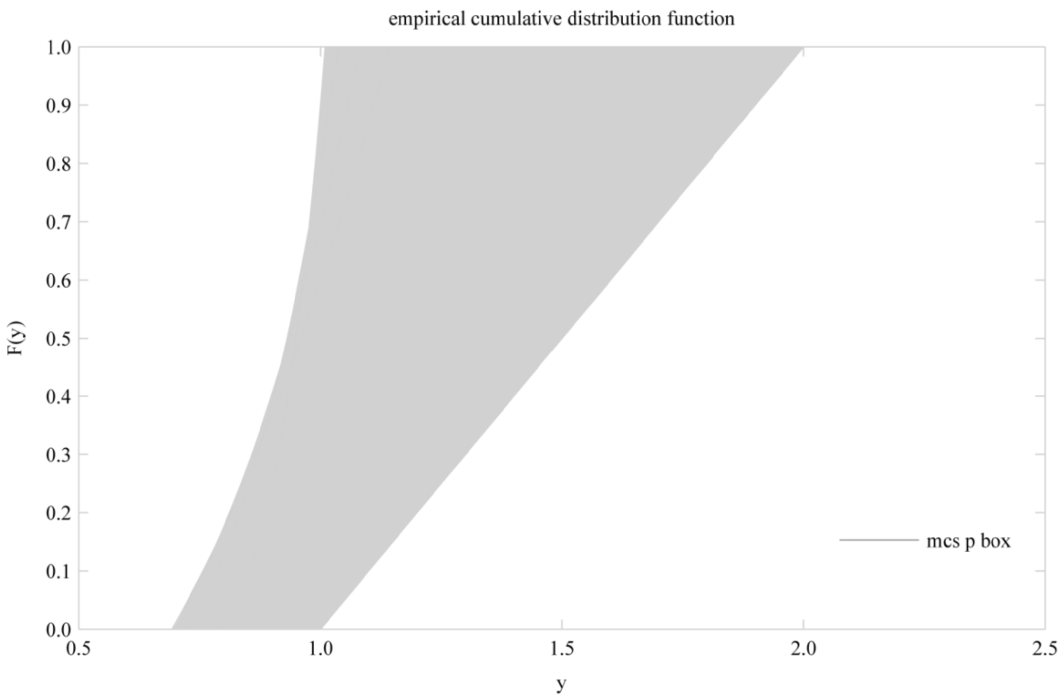

Figure 1.Monte Carlo simulation probability box, mcs p box, envelope overview.

Beer (2009) explains that a model for interval probabilities may be obtained by implementing interval-valued parameters in the description of the probability distribution. Interval probabilities are particularly useful whenever information is insufficient to construct a unique probabilistic model, so that the ensemble of plausible probabilistic models may be included in the analysis of critical cases on the investigation process. Moreover, the computation of system responses reflects a sensitivity analysis wherein different weights may be assigned with respect to the thorough examination of the imprecise probabilistic grid of model cases.

Beer (2009) further describes that fuzzy probabilities gather elements from interval probabilities and evidence theory of belief functions, which combines evidence from different sources and arrives at a degree of belief that takes into account the available information. Under fuzzy probabilities several intervals of different size for the same parameter may be considered in one analysis. This extension may be achieved through fuzzy sets to summarise variants of the parameter interval in one input set which may represent different opinions or just arbitrary intervals to measure the effects of the input interval size on the imprecision of the result.

DESIGN EXAMPLE

The design example is referred to the strip spread foundation on a relatively homogeneous c-φ soil shown in Figure 2, wherein groundwater level is away. Considered the vertical noneccentric loading problem and the calculation model for bearing capacity, the performance function may be described by the simplified Equation (1):

$$M=f(B,D,\gamma_s,c_f,\varphi_f,\gamma_f,P,Q) \qquad (1)$$

if B is the foundation width; D is the soil height above the foundation base; γ_s is the unit weight of the soil above the foundation base; c_f is the cohesion of the foundation soil; φ_f is the friction angle of the foundation soil; γ_f is the unit weight of the foundation soil; P is the dead load; and Q is the live load.

The strip spread foundation is designed by the Eurocode 7 methodology, Design Approach DA.2*. Considered the imprecise probabilistic grid analysis: Table 1 summarises the description of basic input variables, with different distributions; the considered coefficients of correlation between basic input variables are either presented in Table 2. In addition, the imprecise probabilistic fuzzy analysis is performed on uncorrelatedness assumption and Table 3 summarises the description of basic input variables, including both random and interval variables. The coefficient of variation is known from the literature. The imprecise probabilistic grid analysis is performed in a set of scenarios wherein the shear strength parameters of the foundation soil are jointly implemented as intervals, based on which the characteristic values for design are derived and then combined as mean values in the reliability evaluation. The interval model is further combined with the other uncertain parameters, all of them characterised as random variables including dependencies. The imprecise probabilistic fuzzy analysis is performed in a scenario wherein the shear strength parameter friction angle of the foundation soil is separately implemented as interval in the format of a conditional analysis and then combined with other probabilistic parameters, considered a null cohesion of the foundation soil. Thereby, mean values are assigned for the determination of characteristic values for every geotechnical parameter, noted that the characteristic load values are considered as 95% fractile values from the considered normal probability distribution and the remaining parameters are deterministic.

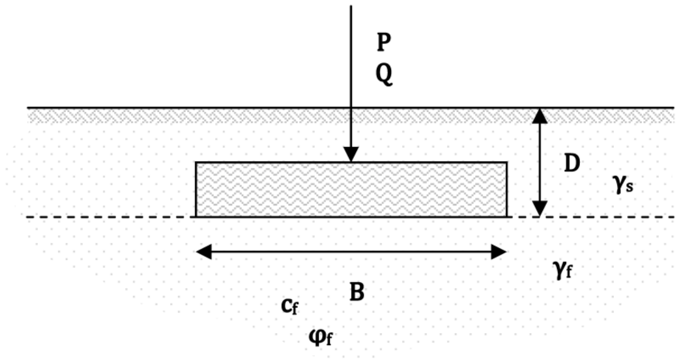

Figure 2.Strip spread foundation.

Table 1.Summary description of basic input variables.

Basic input variables	Distributions	Mean value	Coefficient of variation
B (m)	Deterministic	EC7 DA.2* results	-
D (m)	Deterministic	1.00	-
γ_s (kN/m^3)	Normal	16.80	0.05
c_f (kN/m^2)	Lognormal	[0.00,40.00]	0.40
φ_f (°)	Lognormal	[25.00,40.00]	0.10
γ_f (kN/m^3)	Normal	17.80	0.05
P (kN/m)	Normal	370.00 ∨ 1110.00	0.10
Q (kN/m)	Normal	70.00 ∨ 210.00	0.25

Lower load combination, P (kN/m) = 370.00 ∧ Q (kN/m) = 70.00.
Higher load combination, P (kN/m) = 1110.00 ∧ Q (kN/m) = 210.00.

Table 2.Coefficients of correlation between basic input variables.

$$\begin{bmatrix} \rho_{x1x1} & \rho_{x1x2} & \rho_{x1x3} & \rho_{x1x4} & \rho_{x1x5} & \rho_{x1x6} \\ \rho_{x2x1} & \rho_{x2x2} & \rho_{x2x3} & \rho_{x2x4} & \rho_{x2x5} & \rho_{x2x6} \\ \rho_{x3x1} & \rho_{x3x2} & \rho_{x3x3} & \rho_{x3x4} & \rho_{x3x5} & \rho_{x3x6} \\ \rho_{x4x1} & \rho_{x4x2} & \rho_{x4x3} & \rho_{x4x4} & \rho_{x4x5} & \rho_{x4x6} \\ \rho_{x5x1} & \rho_{x5x2} & \rho_{x5x3} & \rho_{x5x4} & \rho_{x5x5} & \rho_{x5x6} \\ \rho_{x6x1} & \rho_{x6x2} & \rho_{x6x3} & \rho_{x6x4} & \rho_{x6x5} & \rho_{x6x6} \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0 & 0.5 & 0.9 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.0 & 1.0 & 0.5 & 0.0 & 0.0 \\ 0.9 & 0.0 & 0.5 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Correlation matrix

ρ-coefficient of correlation; x_1-γ_s; x_2-c_f; x_3-φ_f; x_4-γ_f; x_5-P; x_6-Q.

Table 3.Summary description of basic input variables.

Basic input variables	Distributions	Mean value	Coefficient of variation
B (m)	Deterministic	EC7 DA.2* results	-
D (m)	Deterministic	1.00	-
γ_s (kN/m^3)	Normal	16.80	0.05
c_f (kN/m^2)	Deterministic	0.00	-
φ_f (°)	Interval	[25.00,40.00]	-
γ_f (kN/m^3)	Normal	17.80	0.05
P (kN/m)	Normal	370.00 ∨ 1110.00	0.10
Q (kN/m)	Normal	70.00 ∨ 210.00	0.25

Lower load combination, P (kN/m) = 370.00 ∧ Q (kN/m) = 70.00.
Higher load combination, P (kN/m) = 1110.00 ∧ Q (kN/m) = 210.00.

RESULTS AND DISCUSSION

At first and considered the Eurocode 7 Design Approach DA.2*, results for the determination of the foundation width B [m] statistics are brought together. Regarding safety, a minimum 0.6 foundation width B [m] is considered from the construction industry practice on the type of geotechnical engineering structure. Thereafter, FORM results are gathered within the specified grid of model cases for the determination of the reliability index β statistics. As illustration, Figure 3 represents FORM results in the reliability index three-dimensional joint view to safety assessment on lower load combination for a 3.0 resistance partial factor considered the interval scenario [0.0,40.0] for cohesion [kN/m^2] and [25.0,40.0] for friction angle [º], median case among the forthcoming cases.

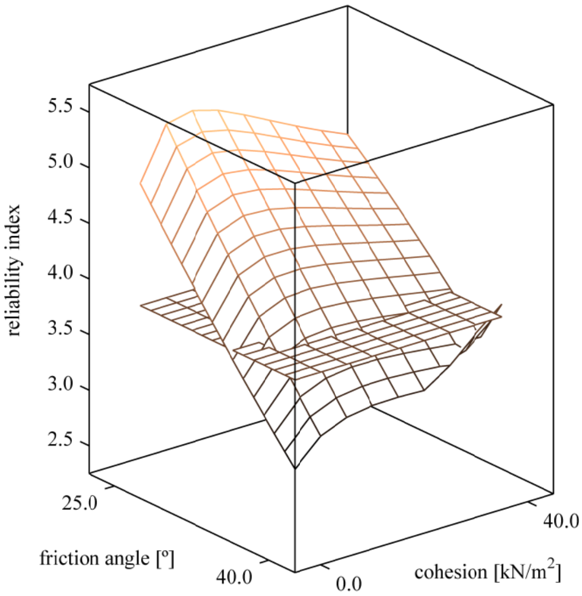

Figure 3.Reliability index three-dimensional joint view to safety assessment on lower load combination for a 3.0 resistance partial factor considered the interval scenario [0.0,40.0] for cohesion [kN/m^2] and [25.0,40.0] for friction angle [º], median case.

In this sketch it is noted the crossed position of the 3.8 target reliability level so that it is observable a curved surface wherein part is below the 3.8 target reliability level, noted that the left corner is the critical and the right corner is shaped due to the minimum 0.6 foundation width B [m]. It is further noted that the maximum reliability index β corresponds to a lower cohesion and to a minimum friction angle characteristic values. Thereby, Figure 4 and Figure 5 represent FORM results for the 3.8 target reliability index ISOLINES, considered the interval scenario [0.0,40.0] for cohesion [kN/m^2] and [25.0,40.0] for friction angle [º], ISOLINES concept detailed hereafter. The individual resistance partial factor is detailed in three cases, 2.5 and 3.0 and 3.5, for the two load combinations, the higher on a 3.0 incremental ratio.

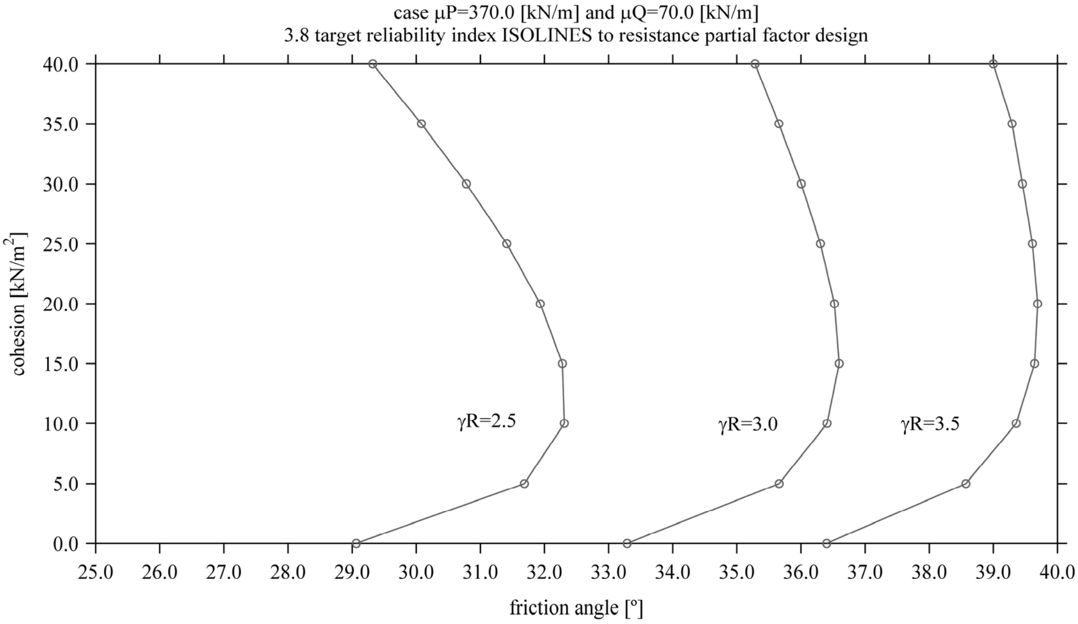

Figure 4.ISOLINES to resistance partial factor design on lower load combination.

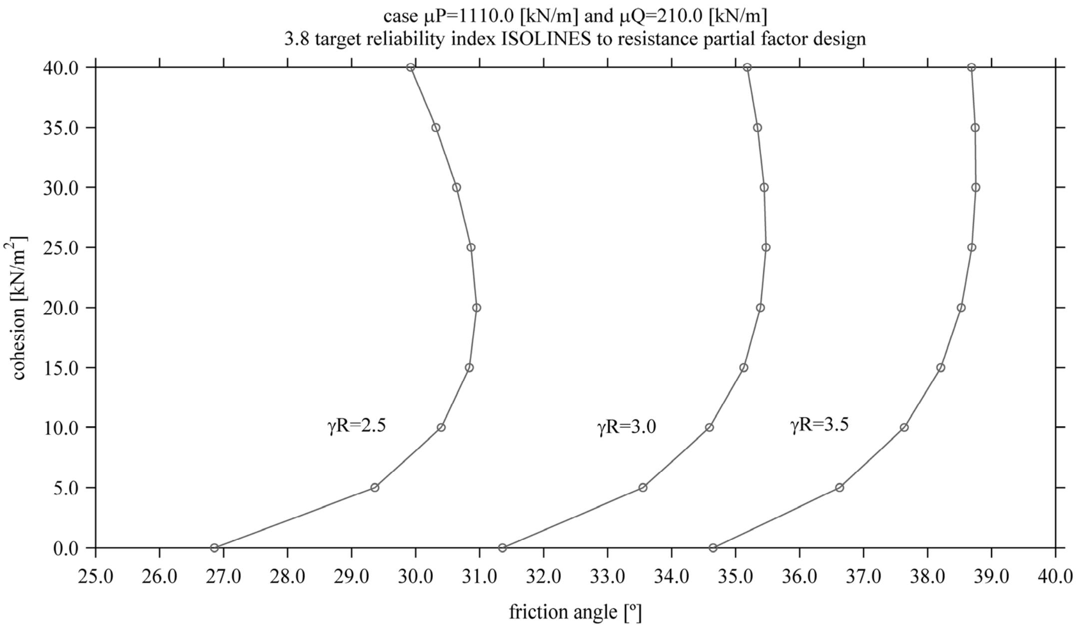

Figure 5.ISOLINES to resistance partial factor design on higher load combination.

Table 4 and Table 5 summarise the statistics for the foundation width B [m] and reliability index β, respectively on lower and higher load combination, noted that the calculation of the weighted resistance partial factor is based on the safe cases in every cluster delimited by each curve drawn respectively at Figure 4 and Figure 5, and not on the safe cases for the calculation of the safe percentage. Furthermore, Table 6 details the resistance partial factor design corresponding to the 3.8 target reliability index calculated from MCS results on four cases corresponding to the lower and higher load combination on 0.0 cohesion [kN/m^2] and 25.0 or 40.0 friction angle [º].

Table 4.Statistics for the foundation width B [m] and reliability index β considered the interval scenario [0.0,40.0] for cohesion [kN/m^2] and [25.0,40.0] for friction angle [º] on lower load combination.

$W\gamma_R$ vs Fs_k	$I\gamma_R$ vs Fs_k	Interval B	Interval β	Safe cases	Safe percentage
3.0 vs 4.1340	2.5 vs 3.4450	[0.6000,3.7821]	[2.6055,4.7294]	059/144	≈ 040
	3.0 vs 4.1340	[0.6000,4.2263]	[2.9966,5.4132]	102/144	≈ 070
	3.5 vs 4.8230	[0.6000,4.6364]	[3.3363,5.9990]	131/144	≈ 090
	4.3 vs 5.9254	[0.6651,5.2377]	[3.8023,6.7906]	144/144	≈ 100

EC7 DA.2* results for the determination of the foundation width B [m] statistics.
FORM results for the determination of the reliability index β statistics.
$W\gamma_R$-weighted resistance partial factor; $I\gamma_R$-individual resistance partial factor.
Fs_k-characteristic safety factor.
$W\gamma_R \approx [2.5\cdot59+3.0\cdot43+3.5\cdot29+4.3\cdot13] / [144] \approx 3.0$.
$Fs_k = \gamma_E\cdot\gamma_R$, γ_E = effect actions partial factor = 1.3780.

Table 5.Statistics for the foundation width B [m] and reliability index β considered the interval scenario [0.0,40.0] for cohesion [kN/m^2] and [25.0,40.0] for friction angle [º] on higher load combination.

$W\gamma_R$ vs Fs_k	$I\gamma_R$ vs Fs_k	Interval B	Interval β	Safe cases	Safe percentage
3.2 vs 4.4096	2.5 vs 3.4450	[1.0726,7.2200]	[2.4658,4.4920]	048/144	≈ 035
	3.0 vs 4.1340	[1.2469,8.0020]	[2.8347,5.1364]	092/144	≈ 065
	3.5 vs 4.8230	[1.4127,8.7221]	[3.1551,5.6880]	119/144	≈ 085
	4.7 vs 6.4766	[1.7825,10.2659]	[3.7897,6.7727]	144/144	≈ 100

EC7 DA.2* results for the determination of the foundation width B [m] statistics.
FORM results for the determination of the reliability index β statistics.
$W\gamma_R$-weighted resistance partial factor; $I\gamma_R$-individual resistance partial factor.
Fs_k-characteristic safety factor.
$W\gamma_R \approx [2.5\cdot48+3.0\cdot44+3.5\cdot27+4.7\cdot25] / [144] \approx 3.2$.
$Fs_k = \gamma_E\cdot\gamma_R$, γ_E = effect actions partial factor = 1.3780.

Table 6.Resistance partial factor design corresponding to the 3.8 target reliability index.

Resistance partial factor vs Characteristic safety factor			
0.0 cohesion [kN/m^2] and 25.0 friction angle [º]		0.0 cohesion [kN/m^2] and 40.0 friction angle [º]	
Lower load combination	Higher load combination	Lower load combination	Higher load combination
2.1514 vs 2.9646	2.3423 vs 3.2277	4.2985 vs 5.9233	4.7245 vs 6.5104

MCS results from 5e6 simulation steps.
$Fs_k = \gamma_E\cdot\gamma_R$, γ_E = effect actions partial factor = 1.3780.

The ISOLINES in Figure 4 and Figure 5 represent the set of shear strength parameters that is capable to satisfy the required 3.8 target reliability index for every resistance partial factor in detail, noted that every combination of shear strength parameters on the left of each curve falls on the safe side. Thus, whenever a variety of possibilities instead of one clear model are advanced, a grid analysis may be pursued to identify a relative importance within a grid of model cases in a decision-making technique which reflects the sensitivity with respect to the load and resistance design. It is noted that this particular case study is based on the assumption that every grid element is eligible on uniform possibility on weighting.

The shear strength parameters on the grid area may be combined and weighted for differentiation through the segmentation of sets into new subsets, analysed any background information on the ground nature and considered the risk tolerance of the geotechnical engineer, as well as any other economic issues related to design feasibility. Complementary knowledge may be provided by a number of references on foundation engineering and geotechnical site investigation or handbooks of design tables, see for instance Look (2007). It is noted that this particular case study is calculated for demonstration only for normal variabilities.

Considered the Eurocode 7 Design Approach DA.2*, results for the determination of the foundation width B [m] for the critical case which considers the pair 0.0 cohesion [kN/m^2] and 40.0 friction angle [º] are determined for resistance partial factors of 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0 on lower and higher load combination. Thereafter, FORM results are gathered within the interval scenario [25.0,40.0] for friction angle [º] for the determination of the reliability index β statistics. Thereby, Figure 6 and Figure 7 represent FORM results for the imprecise probabilistic fuzzy-based approach wherein the reliability index interval is related to the friction angle subsets for every resistance partial factor.

On this fuzzy reliability calculation the friction angle is considered a deterministic parameter combined with the other uncorrelated probabilistic variables. The smooth curved lines at Figure 6 and Figure 7 denote the friction angle demand to attain the indicative reliability marks. They are sketched for every resistance partial factor, so that the assignment of the critical case which considers the pair 0.0 cohesion [kN/m^2] and 40.0 friction angle [º] corresponds to a high reliability space. It is remarked that each line is developed for a unique foundation width B [m] design, the closeness of the case 1.0 at Figure 6 due to the minimum 0.6 foundation width B [m].

Additional FORM constraints to relate interval and probabilistic variables are expendable on uncorrelatedness assumption as by principle it is not possible to attain precision on a verifiable basis on the big data analysis on a high reliability space. The foundation width B [m] is now recalculated for the 1.4 Eurocode 7 resistance partial factor on every case corresponding to the 0.0 cohesion [kN/m^2] and fuzzy interval scenario [25.0,40.0] for friction angle [º], see on Figure 8 FORM results on lower and higher load combination. Figure 9 presents another application of the imprecise probabilistic fuzzy-based approach, utilised to evaluate the minimum foundation width [m] which satisfies the 3.8 target reliability index on lower load combination.

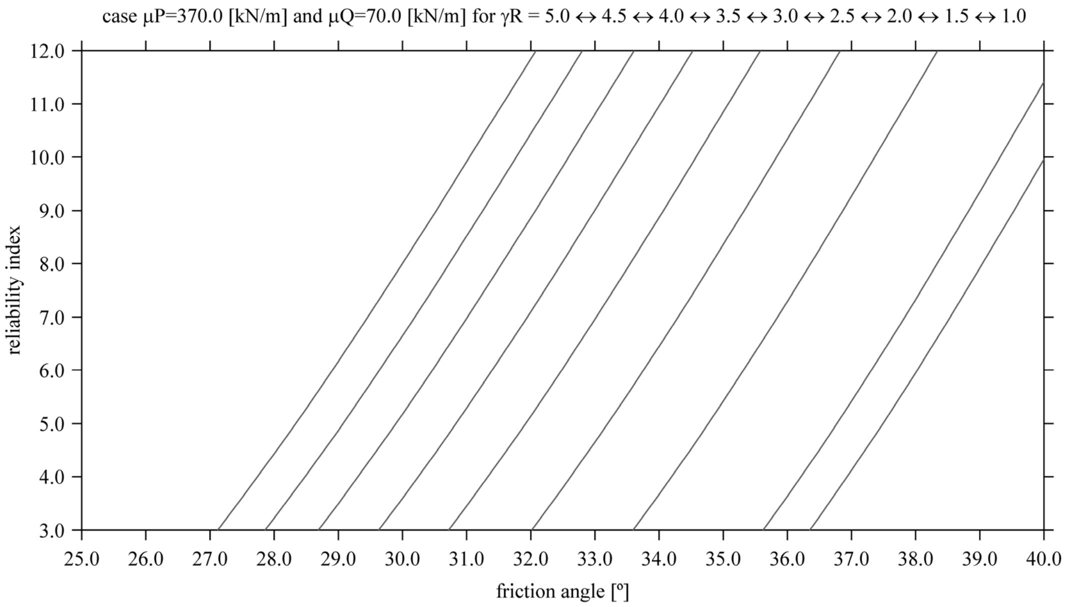

Figure 6.Imprecise probabilistic fuzzy-based approach on constant foundation width [m] for each resistance partial factor level of 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0 on lower load combination.

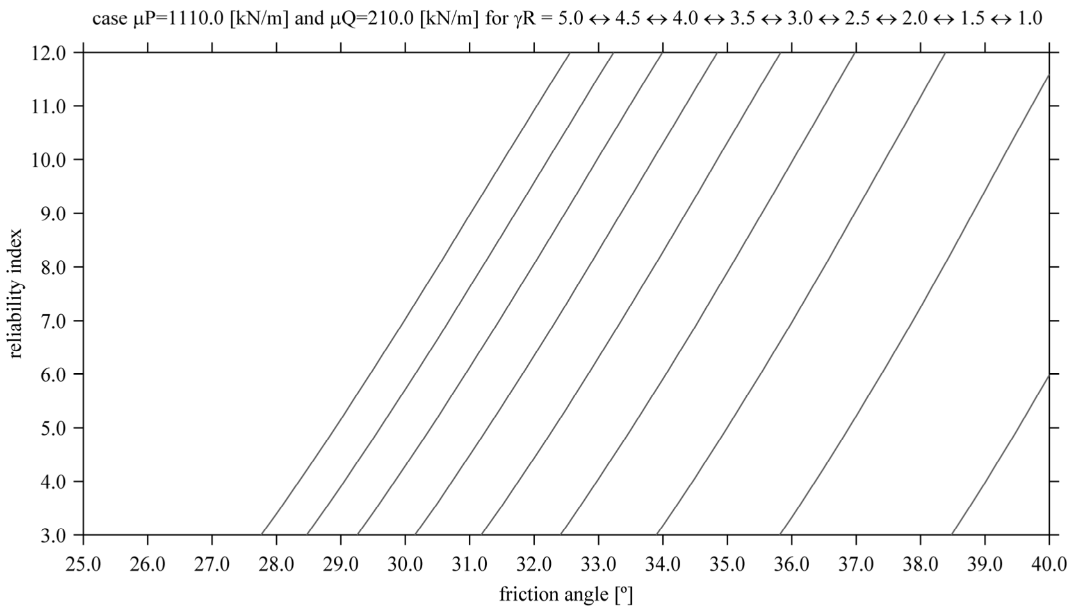

Figure 7.Imprecise probabilistic fuzzy-based approach on constant foundation width [m] for each resistance partial factor level of 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0 on higher load combination.

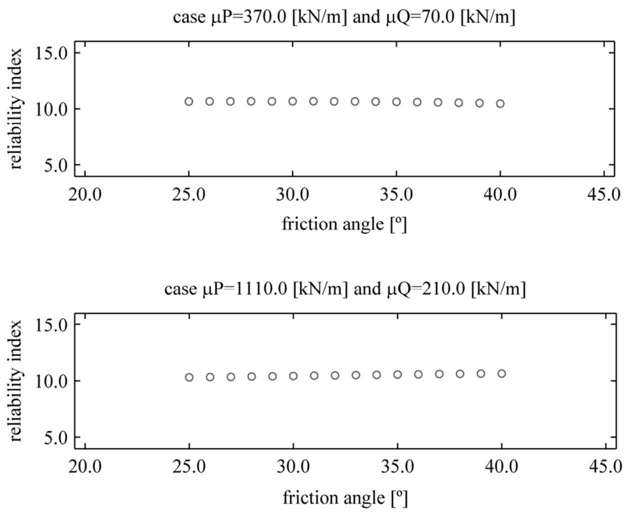

Figure 8.Imprecise probabilistic fuzzy-based approach on variable foundation width [m] for the Eurocode 7 resistance partial factor level of 1.4 on lower and higher load combination.

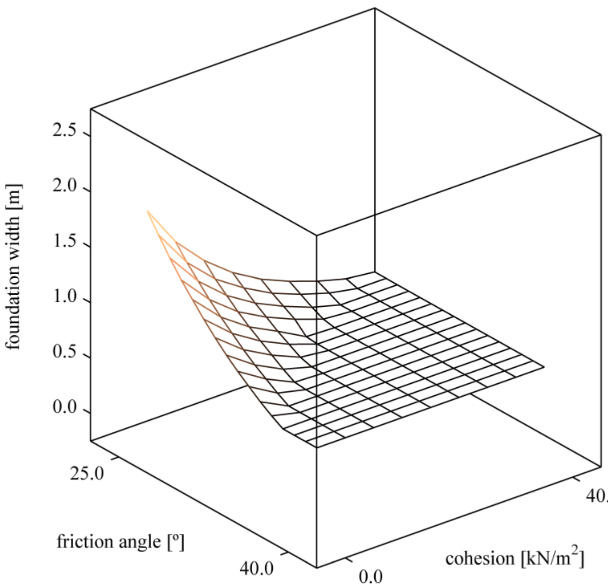

Figure 9.Foundation width [m] three-dimensional joint view to safety assessment on interval scenario [0.0,40.0] for cohesion [kN/m^2] and [25.0,40.0] for friction angle [º] for the Eurocode 7 resistance partial factor level of 1.4 on lower load combination.

Thereafter, the fuzzy reliability calculation evinces now a 10.0 quasi uniform reliability level on Figure 8. It is further noticeable that the minimum 0.6 foundation width B [m] is derived from FORM results in most part of the grid on Figure 9 when considered as deterministic the shear strength parameters. Thereby, the limit state description may compromise a realistic reliability-based analysis and design.

CONCLUSION

On the imprecise probabilistic grid-based approach multiple resistance partial factors may be clearly related to the determination of characteristic values, primary cause of inconsistent reliability evaluations. Expressed simply by intervals, geotechnical parameters on scarce probabilistic information are assigned based on experience.

From the imprecise probabilistic grid analysis applied to this particular case study on lower and higher load combination, a safe percentage between 70 and 75 is attained whenever considered a weighted resistance partial factor calculated on uniform possibility. Thus, a meaningful interpretation on a high dimensional space is based on the joint analysis of multiple cases instead of a lower and upper probabilistic evaluation, noted that the reliability index is variable within a limited expectation.

From the imprecise probabilistic fuzzy analysis applied to this particular case study on lower and higher load combination, the safety concept emerges realigned to a new vision on a quasi uniform reliability level highly attained along the friction angle interval, whenever the shear strength parameters are truly assigned. Thus, it is evinced the capability to approach a uniform reliability level on the parametric range of interest, considered the extension of geotechnical robustness to partial factor design.

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