Annex 2 (by S. Oguzhan Akbas):

Model Uncertainty of Bearing Capacity & Settlement Analysis for Shallow Foundations on Cohesionless Soils

Following summary is based on several published studies:

1. **Bearing Capacity of Shallow Foundations on Cohesionless Soils (summary from Akbas, S.O. & Kulhawy, F.H., *"Axial Compression of Footings in Cohesionless Soils. II: Bearing Capacity*", ASCE J. of Geotech. Geoenv. Eng., 135(11), 2009)**

An extensive database of full-scale field load tests was used to examine the bearing capacity for footings in cohesionless soils. This database summarizes published case histories from 37 sites with 167 axial compression field load tests on footings conducted in cohesionless soils ranging from silt to gravel. The case histories were categorized into three groups based on the quality of the load test data. The range and mean of the geometrical properties and effective stress friction angles, and the available in situ test results, are summarized in Table 1.

Table 1. Summary of Some Geometrical and Geotechnical Properties and Availability of In Situ Test Results

		Footing width B (m)		Footing length L (m)		Footing depth D (m)		$\bar{\phi}_{tc}$		In situ tests			
Data group	No. load tests	Mean	Range	Mean	Range	Mean	Range	Mean	Range	SPT only	CPT only	SPT and CPT	Neither
1	97	0.70	0.25–2.49	0.74	0.25–2.50	0.22	0.00–1.00	37.6	28.0–55.0	35	3	40	19
2	28	0.86	0.30–3.02	0.93	0.30–3.02	0.19	0.00–1.04	38.1	32.0–50.0	13	—	15	—
3	42	1.05	0.30–4.00	1.06	0.30–4.00	0.20	0.00–2.50	39.9	30.0–53.0	11	—	6	25
All	167	0.81	0.25–4.00	0.85	0.25–4.00	0.18	0.00–2.50	38.1	28.0–55.0	59	3	61	44

Each load test curve was evaluated consistently to determine the interpreted failure load, i.e., bearing capacity, using the L_1-L_2 method. This test value then was compared with the theoretical bearing capacity, computed primarily using the basic Vesić model.

The predicted bearing capacity values (Q_{tcp}) are plotted versus the measured Q_{L2} in Fig. 1 for all of the data. This figure shows that the predicted bearing capacity generally is underestimated for Q_{L2} less than about 1,000 kN , and the difference between Q_{tcp} and Q_{L2} generally increases with decreasing Q_{L2}. This trend suggests a possible relationship between the footing width B and Q_{tcp} /Q_{L2}, or bias = ratio of calculated over measured bearing capacity, since the bearing capacity is expected to increase with B, for soils of comparable strength.

All of the further analyses of the data showed an increase in the ratio of predicted-to-measured bearing capacity with increasing B, up to about B=1 m. The comparisons show that, for footing widths B>1 m, the field results agree quite well with the Vesić predictions.

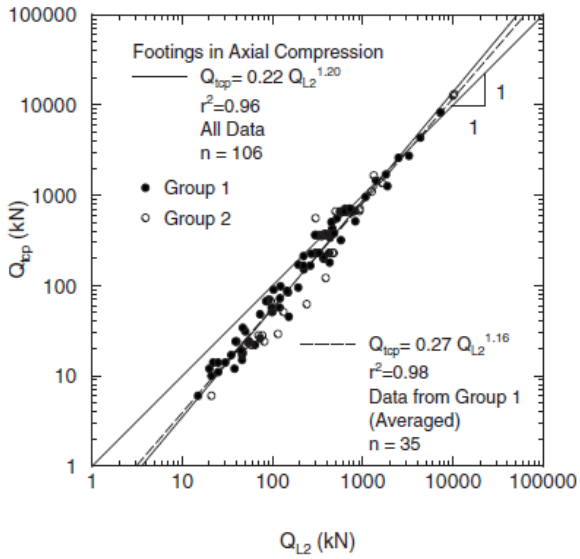

Fig. 1. Predicted (Q_{tcp}) versus measured (Q_{L2}) bearing capacity for footings in drained axial compression (all data and Group 1 data averaged per site)

2. Settlements of Shallow Foundations on Cohesionless Soils

a. Elasticity-based Methods (summary from Akbas, S.O. & Kulhawy, F.H., *"Model Uncertainties in Elasticity-Based Settlement Estimation Methods for Footings on Cohesionless Soils"*, International Symposium on Advances in Foundation Engineering (ISAFE 2013), 2013)

The empirical nature of these various elasticity-based methods suggests a significant uncertainty in the estimated settlements. To assess this uncertainty, an extensive database of 426 case histories was used to assess the model factor (i.e., the ratio of calculated to measured settlement) for the methods developed by D'Appolonia et al. (1970), Parry (1971), Schultze & Sherif (1973), Schmertmann et al. (1978), and Berardi & Lancelotta (1991). Considering the model uncertainty as a random variable, the uncertainty in the model factors is also characterized using the coefficient of variation (COV).

The database includes foundations of various sizes, from small test plates 0.25 m wide to mat foundations up to 135 m wide. The structures corresponding to these foundations include bridges, test footings, buildings, tanks, embankments, chimneys, nuclear reactors, and silos. The details of all available geometric and geotechnical data are provided in Akbas (2007). For those cases where a complete load-settlement curve is available instead of a single response point, the load and settlement values at the elastic limit (L_1), Q_{L1} and ρ_{L1}, were used to estimate the accuracy of the settlement estimation methods. The elastic limit is described elsewhere (e.g., Akbas 2007, Akbas and Kulhawy 2009).

Results for the five selected methods are summarized in Table 1 and include the mean, maximum, minimum, COV, and percent exceedance. It is clear that there is much uncertainty

for all of the methods. The mean ρ_c/ρ_m varies from 1.05 to 2.00, which is a much smaller range than that obtained for Terzaghi & Peck (1948) based methods (Akbas & Kulhawy 2010). Schultze & Sherif (1973) is the most accurate (lowest mean) and the least conservative (underestimates most, in 61% of the cases), while Schmertmann et al. (1978) is the least accurate (highest mean) or most conservative (overestimates most, in 71% of the cases). In general, the results suggest there is a trade-off between accuracy and conservatism of the settlement estimation methods, i.e., as the mean ρ_c/ρ_m approaches one, the number of underestimated cases increases. The COVs, which are indications for statistical precision, range between 70 and 94%, and these high values are consistent with previous research (e.g. Akbas & Kulhawy 2010). These values correspond to a narrow range of standard deviation values between 0.99 and 1.09 for D'Appolonia et al. (1970), Schultze & Sherif (1973), and Parry (1971), while Schmertmann et al. (1978) has the highest standard deviation of 1.40.

Table 1. Relationship between calculated and measured settlements.

Method	(ρ_c/ρ_m)				Exceedance
	Mean	Max.	Min.	COV	$(\rho_c > \rho_m)$
D'Appolonia et al. (1970)	1.45	12.8	0.23	73%	66%
Schultze & Sherif (1973)	1.05	8.34	0.04	94%	39%
Parry (1971)	1.21	8.04	0.09	90%	51%
Schmertmann et al. (1978)	2.00	11.2	0.17	70%	71%
Berardi & Lancelotta (1991)	1.43	6.34	0.12	89%	52%

The following definition is used often to address model uncertainties (e.g. Phoon & Kulhawy 2003):

$$\rho_c = M\rho_m \tag{1}$$

in which M = model factor, which usually is assumed to be a lognormally distributed random variable. The ratios of calculated to measured settlements, or the model factors, were shown in Table 1 for the statistical analysis of the 426 case histories. Using the same database, it was shown by Akbas & Kulhawy (2010) that very slight reduction of model factor COVs are obtained when data from 183 load tests only are used, excluding the remaining 243 observations on instrumented structures. A similar observation was also made by Phoon & Kulhawy (2005) for the model factors that correspond to the lateral capacity of free-head drilled shafts. Therefore, it was assumed that the extraneous uncertainties involved in the estimated model factors would be equal to the corresponding lower end values for measurement errors and inherent variabilities that were specified in Phoon et al. (1995). To incorporate model uncertainties in reliability-based design (RBD), the assumption of lognormality of ρc/ρm or M must be established. For the five settlement methods, ln M is plotted on normal probability plots. From inspection, it was seen that lognormality is a reasonable assumption for the distribution of M. Also, the obtained P values indicate that the

null hypothesis of normality for ln M can not be rejected at a 5% level of significance, except for the D'Appolonia et al. (1970) method.

Further analyses indicate that, the overall accuracy of all methods is low, with no method being capable of estimating settlements within 5 mm of the measured settlements with a probability higher than about 10%.

b. Terzaghi & Peck-based Methods (summary from Akbas, S.O. & Kulhawy, F.H., *"Model Uncertainties in "Terzaghi and Peck" Methods for Estimating Settlement of Footings on Sand"*, GeoFlorida 2010: Advances in Analysis, Modeling & Design (GSP 199), 2010)

The Terzaghi and Peck (1948) method was the first for predicting the settlement of footings on sand using standard penetration test blow counts (N values). Over the following years, various modifications to this basic method were suggested. An extensive database of 426 settlement case histories is used to assess the model factors for this family of methods.

As in many previous studies, the methods of estimating settlements are evaluated first by comparing the calculated (ρ_c) and measured (ρ_m) settlements. Statistically, an accurate method would be one that yields a mean close to or equal to 1.0 for a set of values of the ratio of measured to calculated settlements or vice-versa. The relationships between the calculated and measured settlements for the six selected methods are summarized in Table 1 and include the mean, maximum, minimum, coefficient of variation (COV), and percent exceedance. It is clear that there is much uncertainty for all of the methods. The mean calculated to measured settlement (ρ_c/ρ_m) varies from 1.36 to 3.67. Gibbs and Holtz (1957) is the most accurate (lowest mean), while Terzaghi and Peck (1948) is the least accurate (highest mean) or most conservative (overestimates most, in 93% of the cases). The least conservative methods are those of Gibbs and Holtz (1957) and Alpan (1964), which both overestimated 58% of the cases. In general, the results indicate that there is a trade-off between the accuracy and the conservatism of the settlement estimation methods, i.e., as the mean (ρ_c/ρ_m) approaches one, the number of underestimated cases increases. The same observation was made by Tan and Duncan (1991).

Table 1. Relationships between calculated and measured settlements

Method	(ρ_c/ρ_m)				Exceedance
	Mean	Max.	Min.	COV	($\rho_c > \rho_m$)
Terzaghi & Peck (1948)	3.67	19.4	0.13	69%	93%
Gibbs & Holtz (1957)	1.36	9.78	0.11	69%	58%
Alpan (1964)	1.67	16.1	0.13	102%	58%
Meyerhof (1965)	2.34	13.9	0.14	95%	71%
Peck & Bazaraa (1969)	1.53	8.06	0.10	76%	62%
Peck et al. (1974)	2.70	14.3	0.22	103%	86%

As shown, the COVs range from 69 to 103%. These high values are consistent with previous research (e.g., Jeyapalan and Boehm 1986). Note that the Gibbs and Holtz (1957) and Terzaghi and Peck (1948) methods have the lowest COV (most precise), with a value of 69%. The Peck et al. (1974) and Alpan (1964) methods have the highest COVs (least precise), at 103 and 102%, respectively.

Because of the inherent variability of soil properties, construction variabilities, and measurement errors, it is found reasonable to assume that the model factor statistics are lumped statistics that include the model uncertainties and some extraneous uncertainties, especially for the instrumented structures and, to a lesser extent, for the load tests. To test this argument, a simple comparison was made between the model factors from the load tests only, which includes 183 tests, and from the whole database. The results are shown in Table 2. Except for the Alpan (1964) method, the uncertainties decreased slightly when only data from the load tests were considered.

Table 2. Comparison of model factors for load tests and whole database

Method	Load Tests Only (n = 183)		All Data (n = 426)	
	$M = \rho_c/\rho_m$	COV_M	$M = \rho_c/\rho_m$	COV_M
Terzaghi & Peck (1948)	3.70	66%	3.67	69%
Gibbs & Holtz (1957)	1.40	62%	1.36	69%
Alpan (1964)	1.65	103%	1.67	102%
Meyerhof (1965)	2.51	88%	2.34	95%
Peck & Bazaraa (1969)	1.54	73%	1.53	76%
Peck et al. (1974)	2.62	96%	2.70	103%

The decrease in COV ranges from 3 to 7%. Therefore, when using a database that includes load tests (field and lab) and field measurements on actual structures, extraneous uncertainties should be considered in estimating model uncertainties for RBD calculations. However, considering the Phoon and Kulhawy (2005) results, the effect of normalization, and the likelihood that the equipment and procedural control in documented and monitored cases would be higher than those in average construction, it can be assumed that the effect of extraneous uncertainties on the estimated model factors should be minimal, especially when only the highest quality data are used. This assumption also is supported by the relatively small differences in the COV values in Table 2. Therefore, it was deducted that the extraneous uncertainties involved in the estimated model factors would be equal to the corresponding lower end values for measurement errors and inherent variabilities that were specified in Phoon et al. (1995). It was also determined that for each method, model factors can be effectively modeled as lognormal random variables.

Based on the properties of the lognormal distribution, an attempt was made to estimate model factors that are free from extraneous uncertainties. Then, for a design settlement of 25 mm, the model factors were used to calculate the probability of failure, the reliability index (β), and the probability that the measured settlement will be between 0 and 10 mm, 10 and 25 mm,

and 20 and 25 mm for the six methods considered, using the best-case and worst-case scenarios, respectively. The results indicate that the probability that the settlements would be within 5 mm of the targeted settlement ranges between only about 4 and 12%, even for the best-case scenario. A comparison of the estimated probabilities and reliability indices for the best-case and worst-case scenarios shows that the uncertainties from the N value and loading have only minor effect on the results compared to the model uncertainties.

References:

Akbas, S.O. (2007). Deterministic and probabilistic assessment of settlements of shallow foundations in cohesionless soils. *PhD Thesis*, Cornell Univ, Ithaca, NY.

Akbas, S.O. & Kulhawy, F.H. (2009). "Axial Compression of Footings in Cohesionless Soils. II: Bearing Capacity", ASCE J. of Geotech. Geoenv. Eng., 135(11).

Akbas, S.O. & Kulhawy, F.H. (2010). "Model Uncertainties in "Terzaghi and Peck" Methods for Estimating Settlement of Footings on Sand", *GeoFlorida 2010:Advances in Analysis, Modeling & Design (GSP 199)*, ASCE, 2093-2102.

Akbas, S.O. & Kulhawy, F.H. (2013). "Model Uncertainties in Elasticity-Based Settlement Estimation Methods for Footings on Cohesionless Soils", *International Symposium on Advances in Foundation Engineering (ISAFE 2013)*,

Alpan, I. (1964). "Estimating settlements of foundations on sands." *Civil Engrg & Public Works Rev.*, 59 (700): 1415-1418.

Berardi, B. & Lancelotta, R. (1991). "Stiffness of Granular Soils from Field Performance", *Geotechnique*, 41(1), 149-157.

D'Appolonia, D. J., D'Appolonia, E. D. & Brissette, R. F. (1970). Closure to "Settlement of Spread Footings on Sand", *J. Soil Mech. & Fndns. Div.*, ASCE, 94(SM3), 1011-1053.

Gibbs, H.J. & Holtz, W.H. (1957). "Research on determining the density of sands by spoon penetration testing." *Proc. 4th Intl. Conf. Soil Mech. & Fndn Eng.*, Vol. 1, London: 35-39.

Jeyapalan, J.K. & Boehm, R. (1986). "Procedures for predicting settlements in sands." *Proc. Settlement of Shallow Fndns on Cohesionless Soils (GSP 5)*, Ed. W. O. Martin, ASCE, New York: 1-22.

Meyerhof, G.G. (1965). "Shallow foundations." *J. Soil Mech. & Fndns Div. (ASCE)*, 91(SM2): 21-31

Oweis, I. S. (1979). "Equivalent Linear Model for Predicting Settlements of Sand Bases", *J. Geotech. Eng. Div.*, ASCE, 105(GT12), 1525-1544.

Parry, R. H. G. (1971). "A Direct Method of Estimating Settlements in Sands from SPT Values", *Proc. Symp. Interaction of Structure & Foundation*, Birmingham, 29-37.

Peck, R.B. & Bazaraa A.R.S.S. (1969). “Discussion on settlement of spread footings on sand.” *J. Soil Mechanics and Foundations Div.(ASCE),* 95(SM3): 905-909.

Phoon, K.K., Kulhawy, F.H. & Grigoriu, M.D. (1995). “Reliability-based foundation design for transmission line structures.” *Rpt TR-105000*, EPRI, Palo Alto, 412 p.

Phoon, K.K. & Kulhawy, F.H. (2003). “Evaluation of model uncertainties for reliability-based foundation design.” *Proc. 9th Intl. Conf. Appl. Stat. & Prob. in Civil Eng.*, San Francisco, 1351-1356.

Phoon, K.K. & Kulhawy, F.H. (2005). ”Characterization of model uncertainties for laterally loaded rigid drilled shafts.” *Geotechnique,* 55 (1): 45-54.

Schmertmann, J. H., Hartman, J. P. & Brown, P. R. (1978). “Improved Strain Influence Factor Diagrams”, *J. Geotech. Eng. Div.*, ASCE, 104(GT8), 1131-1135.

Schultze, F. & Sherif, G. (1973). “Prediction of Settlements from Evaluated Settlement Observations for Sand”, *Proc. 8th Intl. Conf. Soil Mech. & Fndn. Eng.* (3), Moscow, 225-230.

Tan, C.K. & Duncan, J.M. (1991). “Settlement of footings on sands: accuracy & reliability.” *Geotech. Eng. Congress 1991 (GSP27),* ASCE, New York: 446-455.

Terzaghi, K. & Peck, R.B. (1948). *Soil Mechanics in Engineering Practice.* Wiley, New York, 566 p.