Annex 3 (by G. Vessia):

Existing works on model uncertainty evaluation for shallow foundation settlement prediction on sandy soils and a new proposal

The estimation of the model uncertainty in the calculation of shallow foundation settlements on sandy soils was first addressed by Terzaghi and Peck (1967) that posed that density and compressibility of sandy deposits is often that erratic to be unrealistic to imagine that any method would be capable of estimating actual settlements of footings. Terzaghi and Peck (1967) suggested that, if several identical footings, all equally loaded, were built on the same sand deposit, the ratio between the upper bound and the lower bound of the estimated settlements will be likely to get to 2 or higher values.

Starting from the preceding considerations, the model uncertainty is considered as one value including other sources of uncertainties like (Uzielli and Mayne 2012):

1) inherent complexity and uniqueness of geomaterials, foundations and their functional interaction;

2) the epistemic uncertainty related to imperfect measurement of the soil-foundation system;

3) the transformation uncertainty in the load-displacement model itself.

A few authors devoted explicit efforts to calculate the <u>model uncertainty</u> in settlement predictions on sandy soils.

Tan and Duncan (1988) introduced the ratio R obtained as calculated settlements S_{calc} divided by measured settlements S_{mis}:

$$R = \frac{S_{calc}}{S_{mis}} \tag{1}$$

Thus databases of R values can be collected. Nevertheless some statistics are needed to rank the magnitude of model uncertainty of several settlement prediction equations for sandy soils S_{calc}.

Then, Tan and Duncan (1988) introduced two quantities, namely *Accuracy* and *Reliability*. They defined the **accuracy** as the average value of this ratio for all the cases in the database. A value of this average equal to unity represents the best possible accuracy.

The **reliability** is defined as the percentage of the cases for which R is equal or greater than 1. Perfect reliability is equal to 100. Finally they used adjustment factors to rank the model uncertainty as a list of ranks from the most to the less efficient settlement prediction equations.

In the same period, Briaud and Tucker (1988) introduced a concise index that took into account **accuracy** and **precision**, namely "Ranking Index" RI:

$$\mathrm{RI} = \mu\left|\left[\ln(\mathrm{R})\right]\right| + \mathrm{s}\left[\ln(\mathrm{R})\right] \tag{2}$$

where μ and s are the mean and the standard deviation of R. The **precision** represents the dispersion of R values about its mean and the **accuracy** measures the greater or smaller closeness of a set of measures to the real value. This latter is an indicator of the central trend, as the mean. Later on, Cherubini and Orr (2000) suggested to use the "Ranking Distance" RD instead of the RI:

$$RD = \sqrt{[1-\mu(R)]^2 + [s(R)]^2} \tag{3}$$

Plotting RD on a Cartesian plane it provides with a geometrical interpretation of the best combination between accuracy and precision among many possible R values (as shown in Figure 1).

Afterwards, Cherubini and Vessia (2009) studied the model uncertainty R of 9 settlement prediction equations for sandy soils that used N_{SPT} measures. The S_{mis} measured settlements used for calculating R values (Eq. 1) come from Burland and Burbidge (1985) database of 192 values from full-scale shallow foundations of several dimensions set in several sandy soils. Thus for each equation 192 R values have been calculated and 9 R samples have been collected whose statistics have been calculated.

As the settlement prediction equations are concerned, the chosen 9 equations are introduced below (all the references can be found in Cherubini and Vessia 2009), according to the following notations:

-B(m) least width of a rectangular foundation or diameter of a circular foundation;

-D(m) depth of footing embedment below ground surface;

-E(kPa) modulus of soil stiffness;

-H(m) depth of the incompressible soil;

-H_s(m) thickness of the compressible layer below the foundation;

-I influence factor for computing settlement from elasticity theory;

-q(kPa) net increase of the effective pressure at foundation level;

-S_c(mm) calculated settlement;

-N_{SPT} is the number of blow count in the SPT;

-σ'_{v0} (kPa) overburden effective pressure at depth z.

- Terzaghi & Peck (1948)

$$S_c = 3.18 \frac{q}{N_{SPT}} \left(\frac{B}{B + B_0} \right)^2 C_d C_w \tag{A-1}$$

where the depth coefficient C_d is assumed to be 1 and for D=0, 0.75 for D≥B (Jorden, 1977), for 0<D<B it is linearly interpolated as follows (Pasqualini, 1983):

$$C_d = \max\begin{cases} 1 - 0.25\dfrac{D}{B} \\ 0.75 \end{cases}$$

The water table coefficient, C_w, is assumed equal to 1 for $D_w \geq 2B$, to 2 for $D_w \leq 0$, for $0 < D_w < 2B$ it is linearly interpolated as follows:

$$C_d = \max\begin{cases} 2 \quad \text{if } D_w \leq 0 \\ 1 \quad \text{if } D_w \geq 2B \\ 2 - 0.5\dfrac{D_w}{B} \quad \text{otherwise} \end{cases}$$

- Meyerhof (1965)

$$S_c = 2.12\frac{q}{N_{SPT}}\left(\frac{B}{B+B_0}\right)^2 C_d \qquad \text{(B-2)}$$

where C_d has the same meaning as in Terzaghi and Peck's method (1948).

- Meigh and Hobbs (1975)

$$S_c = 3.18\frac{q}{N_{SPT}}\left(\frac{B}{B+B_0}\right)^2 C_d \frac{4}{\dfrac{q_c}{N_{SPT}}} \qquad \text{(C-3)}$$

where for C_d the authors refer to Terzaghi and Peck's method and the ratio q_c/N_{SPT} depends on the grain-size distribution of soil.

- Arnold (1980)

$$S_c = \frac{43.065B\alpha}{\left[1+(3.28B)^m\right]^2} H_S \ln\left(\frac{Q}{Q-0.5q}\right) \qquad \text{(D-4)}$$

H_s being the depth of the compressible layer below the foundation (in m):

$$H_s = \min(2B, H-D)$$

and

$$\alpha = 0.032766 - 0.0002134D_R$$

$$Q = 19.63D_R - 263.3$$

$$m = 0.788 + 0.0025D_R$$

where D_R is the relative density, given by:

$$D_R(\%) = 25.6 + \\ + 20.37\sqrt{\frac{1.26\cdot(N_{SPT}-2.4)}{0.0208\sigma'_{v0,B+D/2}+1.36}-1}$$

This expression is valid for $N_{SPT} \geq 6$ and $45\% \leq D_R \leq 100\%$.

- Burland and Burbidge (1985)

$$S_c = 1.71\cdot\left(q_{gross} - \frac{2}{3}\sigma'_{v0,D}\right) \\ \frac{B^{0.7}}{(N_{SPT})_{AV}^{1.4}}\left(\frac{1.25L}{L+0.25b}\right)^2 \frac{H_s}{Z}\left(2-\frac{H_s}{Z}\right) \qquad \text{(E-5)}$$

Where q_{gross} is the gross bearing pressure at foundation level (kPa), Z is the depth of influence for N_{SPT} averaging that is $(N_{SPT})_{AV.}$ If N_{SPT} values decrease with depth, Z is equal to 2B, otherwise it is calculated by the following interpolating equation:

$$Z = 0.933B^{0.779}$$

- Anagnostopoulos, Papadopoulos and Kavvadas (1991)

These authors suggest two settlement formulas:

Formula1:

$$S_c \begin{cases} 0.57q^{0.94}B^{0.90}N_{SPT}^{-0.87} & \text{for } 0 < N_{SPT} \leq 10 \\ 0.35q^{1.01}B^{0.69}N_{SPT}^{-0.94} & \text{for } 10 < N_{SPT} \leq 30 \\ 604q^{0.9}B^{0.76}N_{SPT}^{-0.82} & \text{for } N_{SPT} > 30 \end{cases} \qquad \text{(F-6)}$$

Formula2:

$$S_c \begin{cases} 1.9\dfrac{q^{0.77}B^{0.45}}{N^{1.08}} & \text{for } B \leq 3\text{m} \\ 1.64\dfrac{q^{1.02}B^{0.59}}{N^{1.37}} & \text{for } B > 3\text{m} \end{cases} \qquad \text{(G-7)}$$

- Schultze and Sherif (1973)

Starting from the general formula for predicted settlement by means of the elasticity equation:

$$S_c = \frac{qB}{E} I \qquad \text{(H-8)}$$

This expression is obtained by numerical integration of Boussinesq solution for circular footing whereas for rectangular footing the influence factor I by Steinbrenner's method (1934) is introduced. The stiffness modulus E (in kPa) is given by:

$$E = 1678(N_{SPT})^{0.87}\sqrt{B}\left(1 + 0.4\frac{D}{B}\right)$$

- Berardi and Lancellotta (1991)

As the preceding authors do, Berardi and Lancellotta start from the elasticity equation for settlement:

$$S_c = \frac{qB}{E} I \qquad \text{(I-9)}$$

but calculate each term differently from the previous authors. As a matter of fact, the influence factor I has been calculated by numerical integration of Boussinesq solution considering rigid footing. The stiffness modulus, E, is given by:

$$E = K_E p_a \sqrt{\frac{\sigma'_{v0,D+\frac{B}{2}} + \Delta\sigma'_{v,D+\frac{B}{2}}}{p_a}}$$

where p_a is the reference pressure and K_E is initially evaluated as:

$$K_E = 100 + 900 D_R$$

$$D_R = \min\left\{1; \sqrt{\frac{N_1}{60}}\right\}$$

$$N_1 = \frac{2}{1 + \frac{\sigma'_{v0,D+B/2}}{98.1}} N_{SPT}$$

Once the settlement has been calculated, K_E is corrected through the equation:

$$K_{E,corr} = K_E \cdot 0.1912 \cdot \left(\frac{S_c}{B}\right)^{-0.6248}$$

and the predicted settlement is recalculated using $K_{E,corr}$ instead of K_E.

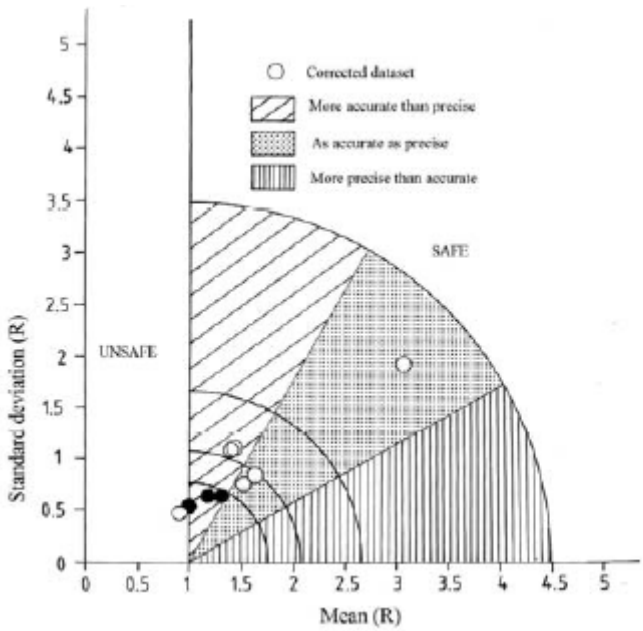

Figure 1. Plane showing both mean values (on the x axis) and standard deviations (on the y axis) of R for the nine settlement formulas.

Cherubini and Vessia (2009) suggested a procedure to rank the 9 R samples of model uncertainty through the use of four indexes:

1) Precision
2) Accuracy
3) Entropy
4) Relative entropy

The first two indexes can be combined into the Ranking distance and plotted as shown in Figure 1.

Entropy (Shannon 1948) and relative entropy (Kinsley 1983), like the variance, are measures of the dispersion according to the following expressions, respectively:

$$H(x) = -\sum_{i=1}^{n} P_i \ln(P_i) \quad (4)$$

$$H_r(x) = \frac{H(x)}{H_{max}} \quad (5)$$

where P_i is the probability value associated with the calculated R value.

The procedure consists on:

(1) calculating the RD, whose ideal value is 0;

(2) calculating the relative entropy by Eq. (5) whose ideal value is about 0.

(3) ranking the equations according the calculated values of RD and H_r.

The final aim of the proposed procedure, that can be extended to a larger number of equations, is to provide a quick tool for designing. In fact, curves of different levels of probability for settlement target values can be drawn through several formulas. Hereafter just to provide with an example, three curves have been calculated for three formulas, that are: two by Anagnostopoulos et al. (1991), named APK1 and APK2 and one by Schultze and Sherif (1973), named SS. The preceding equations are the most precise formulas among the nine considered by Cherubini and Vessia (2009) according to the available full-scale data by Burland and Burbidge's settlement database.

As far as the values of the shallow foundation settlements are concerned, Eurocode 7 (BS 2004, updated 2013) suggests an upper limit equal to 50mm to be accepted. Nevertheless, the preceding value refers to common buildings with no complex structures. In practice, settlements can be accepted if they do not cause both static and functional problems to structures and services. Thus, five values of settlements are hereafter considered as acceptable: 5, 15, 25, 35 and 45mm. The exceeding probability curves are related to three samples of R that are normally distributed. The curves are drawn according to the following expression:

$$p(R < \overline{R}) = p\left(R < \frac{\overline{S_{calc}}}{\overline{S_{mis}}} \right) = p\left(\frac{\overline{S_{calc}}}{R} > \overline{S_{mis}} \right) \qquad (6)$$

This expression shows that the probability that a settlement value is higher than the corresponding measured one is equal to the probability that any value of R is lower than a fixed value of R (5, 15, 25, 35 and 45). Figures 2-4 show the probability curves calculated through APK1, APK2 and SS.

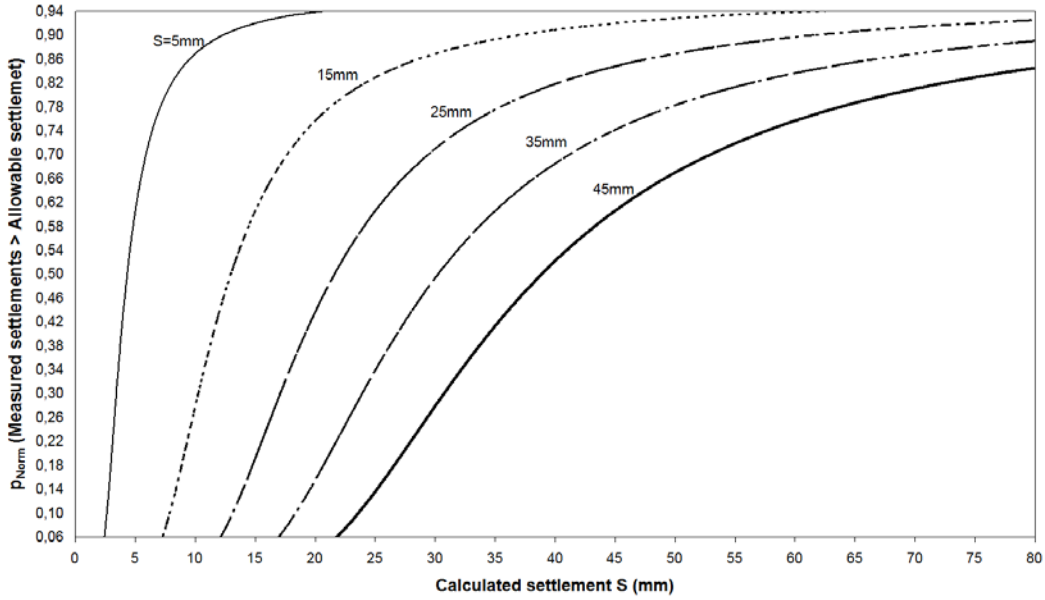

Figure 2. Normal-distribution probability curves corresponding to different values of acceptable settlements calculated by APK1 formula.

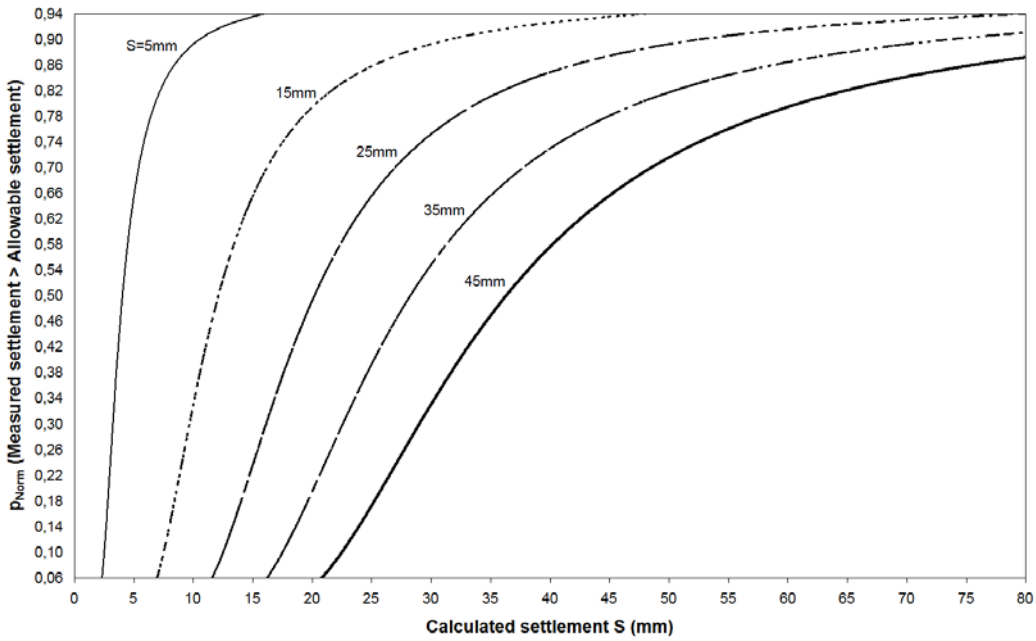

Figure 3. Normal-distribution probability curves corresponding to different values of acceptable settlements calculated by APK2 formula.

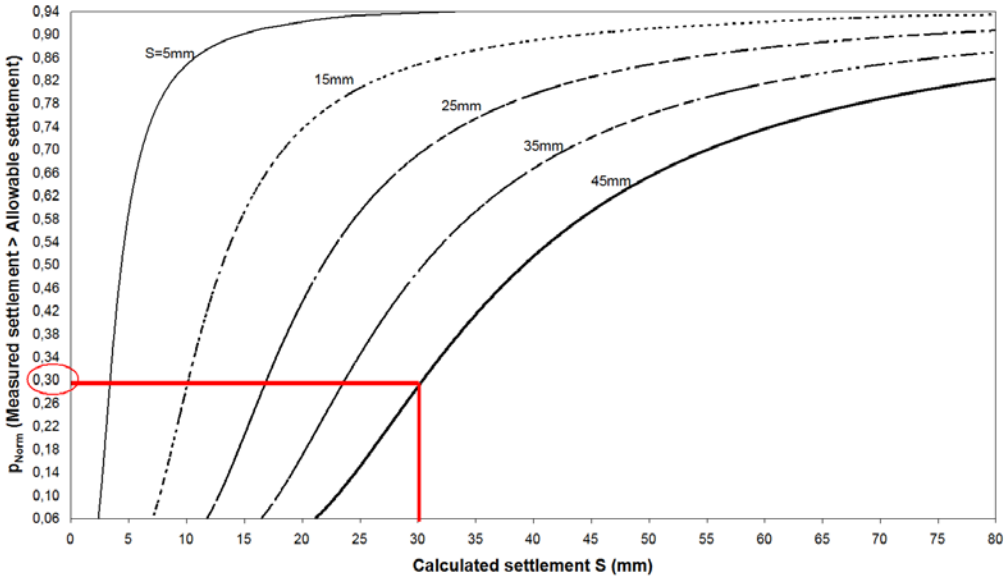

Figure 4. Normal-distribution probability curves corresponding to different values of acceptable settlements calculated by SS formula.

The values corresponding to the probability of exceedance can be obtained from these settlement probability curves. For example, considering that a 30mm settlement has been estimated by means of the SS formula, Figure 4 shows that there is a 30% probability that the actual settlement under footing will be more than 45mm, that is to say the probability of calculating settlements below 45mm will be 70%. Similarly, there will be a 15% probability that the settlement is below 15mm, i.e. an 85% probability that the settlement is more than 15mm. For all three formulas the exceeding probability curves for values of settlements occurring between the curves plotted on the graph will be obtained by interpolating the values of probability for the two curves which mark the range.

These probability charts let the engineer know the probability that a certain value of the settlement measured is exceeded provided that the settlement value is known, i.e. they provide the conditioned probability of the settlements measured:

$$P\left(S_{mis} > \overline{S} \middle| S_{calc} = \overline{s}\right) \tag{7}$$

where S_{calc} and S_{mis} are the calculated and measured settlements, respectively; $\overline{s}$ and $\overline{S}$ are the values of their respective settlements. So the conditioned probability in terms of not-exceedance will be the following:

$$P\left(S_{mis} > \overline{S} \middle| S_{calc} = \overline{s}\right) = 1 - P\left(S_{mis} < \overline{S} \middle| S_{calc} = \overline{s}\right) \tag{8}$$

Therefore, as a result of a reliability-based design in terms of settlements (calculated by means of one of the three formulas APK1, APK2 and SS), the probability that the actual settlement of the footing does not exceed a given value can be easily got by the conditioned probability theorem:

$$P\left(S_{mis} < \overline{S} \middle| S_{calc} \le \overline{s}\right) = P\left(S_{mis} < \overline{S} \middle| S_{calc} = \overline{s}\right) \cdot P\left(S_{calc} \le \overline{s}\right) \tag{9}$$

where the first term comes from the probability charts (Figures 2-4) while the second one derives from the reliability-based design. In fact, if the settlements are estimated by the SS formula, and the probability of not exceedance of a 20mm settlement is equal to 10^{-4}, the probability that the settlement actually measured is below 25mm will be easily got from Figure 4:

$$P\left(S_{mis} < 25mm \middle| S_{calc} \le 20mm\right)_{SS} = \left(1 - 0.46\right) \cdot 10^{-4} = 0.54 \cdot 10^{-4} \tag{10}$$

If the two other formulas are used to estimate settlements the following results will be calculated:

$$P\left(S_{mis} < 25mm \middle| S_{calc} \le 20mm\right)_{APK1} = \left(1 - 0.42\right) \cdot 10^{-4} = 0.58 \cdot 10^{-4} \tag{11}$$

$$P\left(S_{mis} < 25mm \middle| S_{calc} \le 20mm\right)_{APK2} = \left(1 - 0.50\right) \cdot 10^{-4} = 0.50 \cdot 10^{-4} \tag{12}$$

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