Partial Factor Limit States Design and Reliability Based Design: A Practical Comparison:

Nico de Koker (PhD), Peter Day (DEng Pr Eng)
ndekoker@gmail.com*; day@jaws.co.za*
Department of Civil Engineering, University of Stellenbosch, South Africa.

1. INTRODUCTION

1.1 Background

Reliability analysis in structural and geotechnical engineering has been around for many years and provides the basis for the formulation and calibration of limit states design codes, such as the Eurocodes. In the past 10 years, reliability based design methods have become sufficiently accessible to serve as practical tools in a geotechnical design office, rather than being confined to research applications.

A developing country such as South Africa is not bound by any regional economic grouping or other influences to adopt a particular design method or set of codes, as is the case with member states within the European Union and the Eurocodes. Despite the fact that Limit States Design is now the international norm, the perceived obscurity of the method, multiplicity of approaches and openness to interpretation are impediments to widespread adoption of the method. As a result, some practitioners are electing to continue using working stress design methods while, at the other end of the spectrum, others are advocating the use of reliability based design methods.

The purpose of this discussion note is compare the solutions to common geotechnical problems obtained from limit states design with those obtained using working stress design and reliability based design methods. These simple comparisons highlight some the limitations and merits of the various design methods.

1.2 Approach

Shortly after the release of Eurocode 7 (EN1997-1, 2004), an international workshop on the implementation of Eurocode 7 was held at Trinity College in Dublin (Orr, 2005). One of the purposes of this workshop was to compare the solutions to common geotechnical problems as prepared by representatives of various countries in Europe. Its lasting legacy was an appreciation for the diversity of approaches and interpretations of Eurocode 7 and a set of model solutions which have been of considerable value to those seeking to implement the code.

In much the same way, there is benefit in applying reliability based design to simple problems and comparing the solutions obtained with those from other design methods. The examples presented in this discussion document have deliberately been kept simple as this facilitates an intuitive interpretation of the outcomes, unobscured by any complexities in the analysis.

The solutions presented here are based on an interpretation of Eurocode 7 and methods of selection of parameters that can be expected from a typical geotechnical practitioner for routine design.

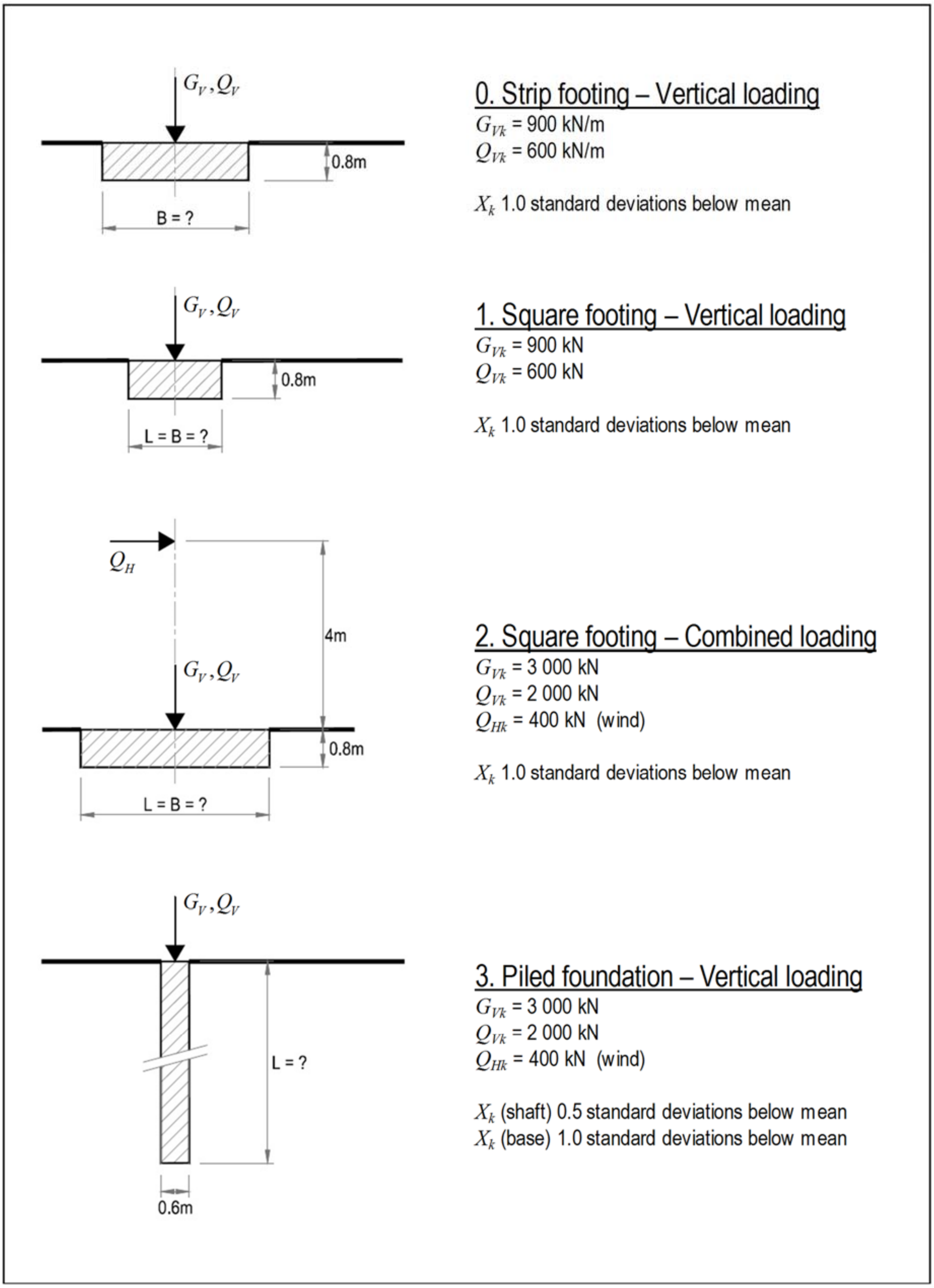


Figure 1a: Examples 0 – 3 (Foundations)

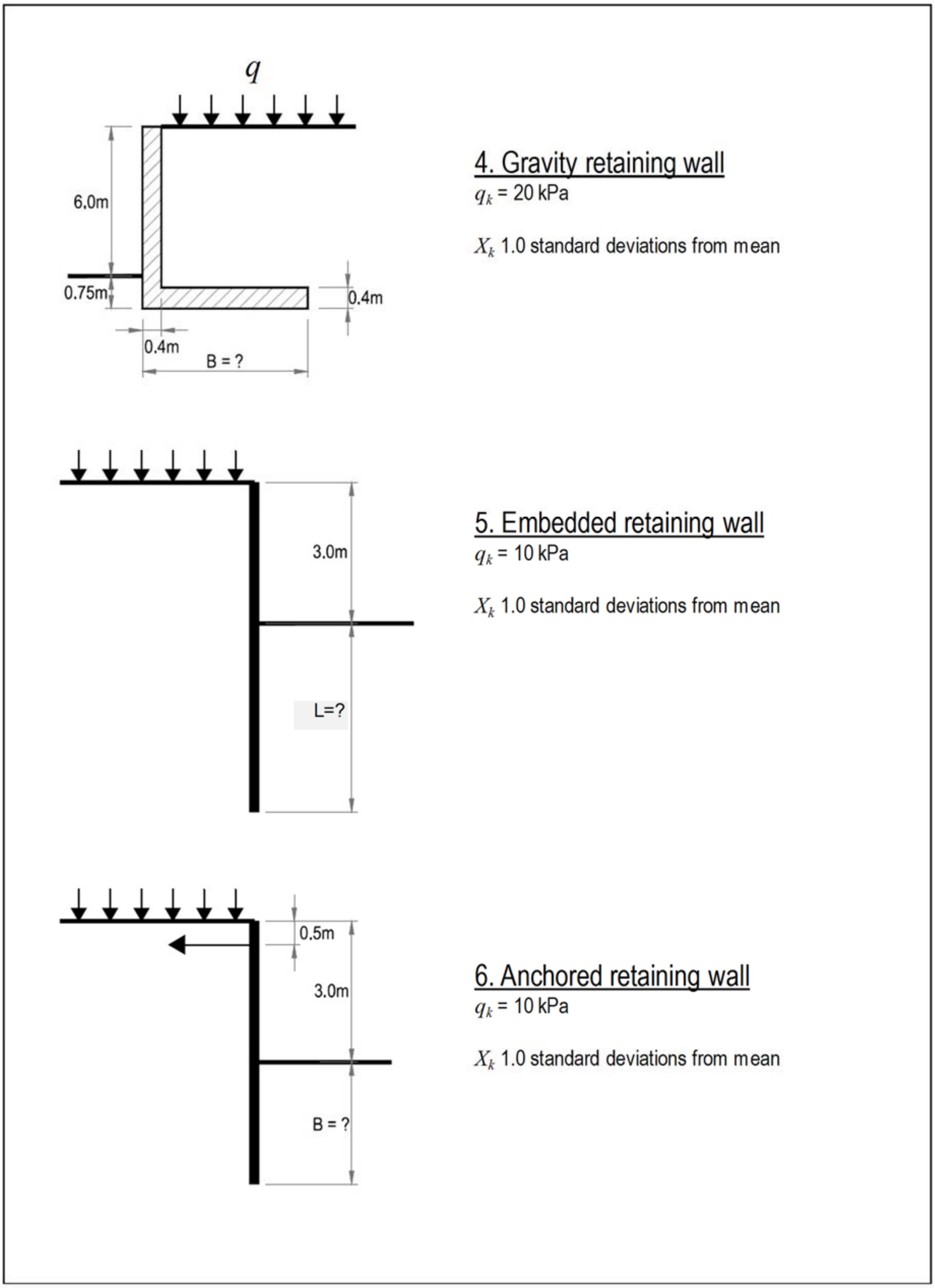


Figure 1b: Examples 4 – 6 (Retaining Structures)

2. METHODOLOGY

2.1 Problems Analysed

The problems analysed were based on those selected for the Dublin Workshop (Orr, 2005) and are shown in Figure 1. Only problems with closed-form solutions were used. In each case, the solution required is a dimension that determines the adequacy of the structure, such as the width of a footing or the length of a pile.

The nominal dimensions shown in Figure 1 were taken as characteristic values with no adjustment.

2.2 Soil Type

For the purposes of this study, a single soil type was chosen, namely a cohesionless sand with a deep water table. As such, there are only two material (soil) parameters to be considered, namely friction angle and density.

Table 1: Assumed soil properties

Soil Property Parameter	Friction angle	Bulk density
Symbol	ϕ'_k	γ_k
Characteristic value	32°	20 kN/m^3
Distribution	log-normal	normal
Coefficient of variation	0.10	0.05
Correlation coefficient	0.2	

In practice, the characteristic value of a material property is determined from the mean value minus (or plus) a number (n) of standard deviations as selected by the designer (Schneider, 1997). The multiplier n depends mainly on the number of test results available and the extent to which the occurrence of the limit state (i.e. failure) is dependent on the average rather than minimum/maximum value of the parameter. In this study, n was taken as 1.0 in all cases except Example 3 where n=0.5 was used for the pile shaft resistance. For the purposes of the reliability analysis, the corresponding mean value is back-figured from the characteristic value by applying the process in reverse.

2.3 Loading

As illustrated in Figure 1, the following loads (actions) have been considered in the examples:

- Permanent action (G_v): Fixed value, mean value = characteristic value.
- Variable action vertical (Q_v): Log-normal distribution, coefficient of variation 0.25.
- Variable action horiz. (Q_h): Gumbel distribution, coefficient of variation 0.5 (wind).

The vertical and horizontal actions are assumed to be independent. An action combination factor of 0.7 has been applied to the accompanying variable action. The vertical variable action may be favourable or unfavourable.

The statistical distributions, coefficients of variation and ratio of characteristic to mean loading is based on Retief & Dunaiski (2009) and Phoon & Kulhawy (1999).

2.4 Analysis

The following approach was followed in this investigation:

i. Find the solution to the problem that satisfies the ultimate limit state verification requirements of EN1990 and EN1997 (Design approach 1, Combination 2) using the characteristic values of loads and material properties.

ii. Determine the mean values of loads and material properties corresponding to the given characteristic values.

iii. Using the corresponding mean values and coefficients of variation, determine the reliability index (β) of the solution using Monte Carlo and FORM.

iv. Determine the factor of safety (FoS) using working stress design methods.

In the case of example 2 (square footing under vertical and horizontal loads), the following additional analyses were performed, both of which required a re-evaluation of the limit state design compliant solution for each new set of material properties:

v. Repeat of steps (i) – (iv) for a range of material properties to examine the variation of β and FoS.

vi. Repeat of steps (i) – (iii) for a range of coefficients of variation of the material properties to explore the sensitivity of β to uncertainties in material properties.

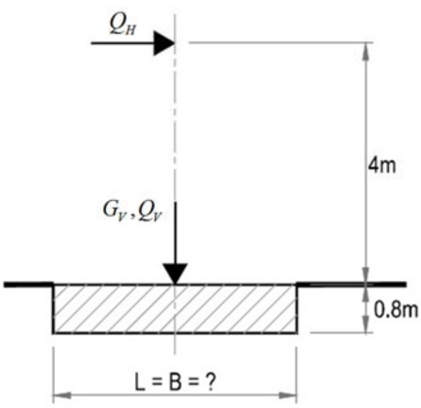
3. EXAMPLE 2: VERTICALLY AND HORIZONTALLY LOADED SQUARE FOOTING

This example, which is based on Example 2 from Orr (2005), will be discussed in detail.

The same methodology was followed for the other examples, for which only a summary of the results is reported in Table 3 of this discussion document.

3.1 Problem Setup

The following problem was analysed.



Characteristic Values	Partial Factors
$G_{V,k} = 3000kN$	$\gamma_G = 1.0$
$Q_{V,k} = 2000kN$	$\gamma_Q = 1.3$
$Q_{H,k} = 400kN$	$\psi_0 = 0.7$
$\phi'_k = 32^o$	$\gamma_\gamma = 1.0$
$\gamma_k = 20.0\,kN/m^3$	$\gamma_\phi = 1.25$

Mean Values

$\bar{G}_V = 3000kN$

$\bar{Q}_V = 1375kN$ $(CoV\ 0.25; \sim Log-Normal)$

$\bar{Q}_H = 207kN$ $(CoV\ 0.50; \sim Gumbel)$

$\bar{\phi}' = 35.55^o$ $(CoV\ 0.10; \sim Log-Normal)$

$\bar{\gamma} = 21.05\,kN/m^3$ $(CoV\ 0.05; \sim Normal)$

$\rho_{\phi,\gamma} = 0.20$

Figure 2: Input data for Example 2

3.2 LSD and WSD solutions

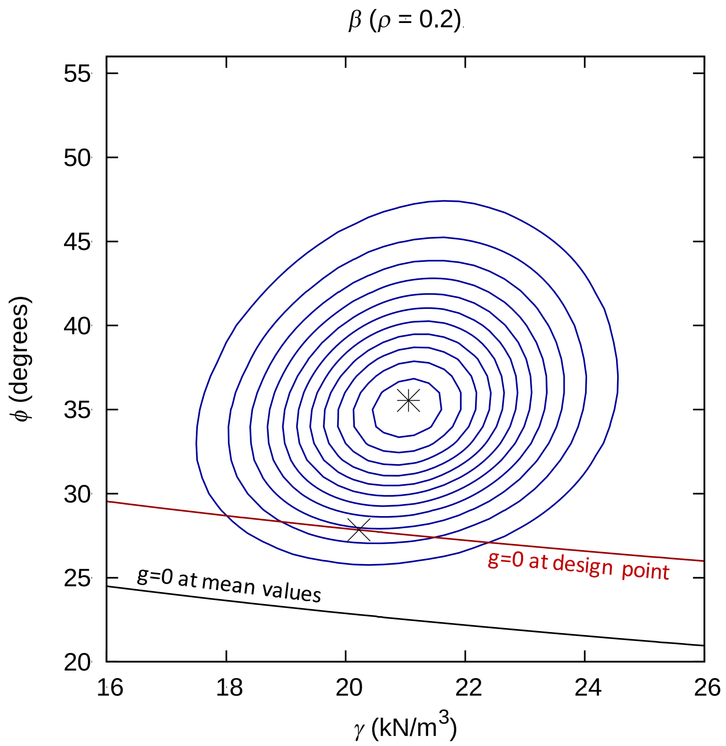
The limit states and working stress design solutions for this example are given below.

LSD Solution	WSD (average values)	WSD (characteristic values)
$B = L$ = 3.99m	$B = L$ = 3.99m	$B = L$ = 3.99m
e_B = 0.49m	e_B = 0.21m	e_B = 0.36m
B' = 3.02m	B' = 3.57m	B' = 3.27m
$R_d = E_d$ = 5 126kN	q_f = 2 067kPa	q_f = 1 031kPa
	FoS = 6.58	FoS = 2.60

3.3 Reliability Analysis

3.3.1 ϕ' ~ Log-normal

The results of the Monte Carlo and FORM analyses of the Eurocode-compliant solution are summarised in Figure 3.



MONTE CARLO
With correlation
β : 3.59
N(fail) : 162
N(iter) : 10^6

FORM
With correlation:
β : 3.69

Without correlation:
β : 3.73

Design Point:
(with correlation, X on figure)

Q_V	:	1306	kN
Q_H	:	638	kN
γ	:	20.22	kN/m^3
ϕ	:	27.85	degrees

Figure 3: Reliability analysis of Eurocode-compliant solution to Example 2

3.3.2 Effect of ϕ' distribution

The effect of different assumptions regarding the statistical distribution of the friction angle of the soil is summarised in Table 2.

Table 2: Effect of assumed statistical distribution for friction angle

	ϕ' ~ Normal	ϕ' ~ Log-normal	$tan\ \phi'$ ~ Log-normal*
Monte Carlo - with correlation ($\rho_{\phi,\gamma}$ = 0.20)			
Reliability Index (β)	3.26	3.59	3.59
No. iterations	10^6	10^6	10^6
No. failures	559	162	168
FORM			
β, $\rho_{\phi,\gamma}$ = 0.20	3.40	3.69	3.69
β, $\rho_{\phi,\gamma}$ = 0.0	3.46	3.73	3.73
Design point, $\rho_{\phi,\gamma}$ = 0.20			
Q_V	1406	1305	1312
Q_H	358	638	622
γ	20.07	20.22	20.02
ϕ'	24.61	27.85	27.63

* $\bar{\phi}'$ adjusted to match FORM β = 3.69. ($\bar{\phi}'$ = 35.41°, CoV = 0.121)

3.4 Effect of range of material properties

The above analysis of Example 2 considered only a single set of ground properties. For partial factor limit states design to be an acceptable method of design, the target level of reliability should be achieved across the range of material properties likely to be encountered in practice.

The effect of variation in the values of the soil properties is shown in Figure 4Figure 5, in which the limit states design solution is evaluated for a range of ϕ'-γ values, and the corresponding level of reliability determined using FORM.

3.5 Effect of variance in material properties

In the same way, the target level of reliability should be achieved for the range in variance (coefficient of variation) of geotechnical parameters likely to occur in practice. Changing the variance of the material properties affects the ratio of the characteristic value to the mean value as described in section 2.2.

The effect of changes in the variance of the soil properties is shown in Figure 5.

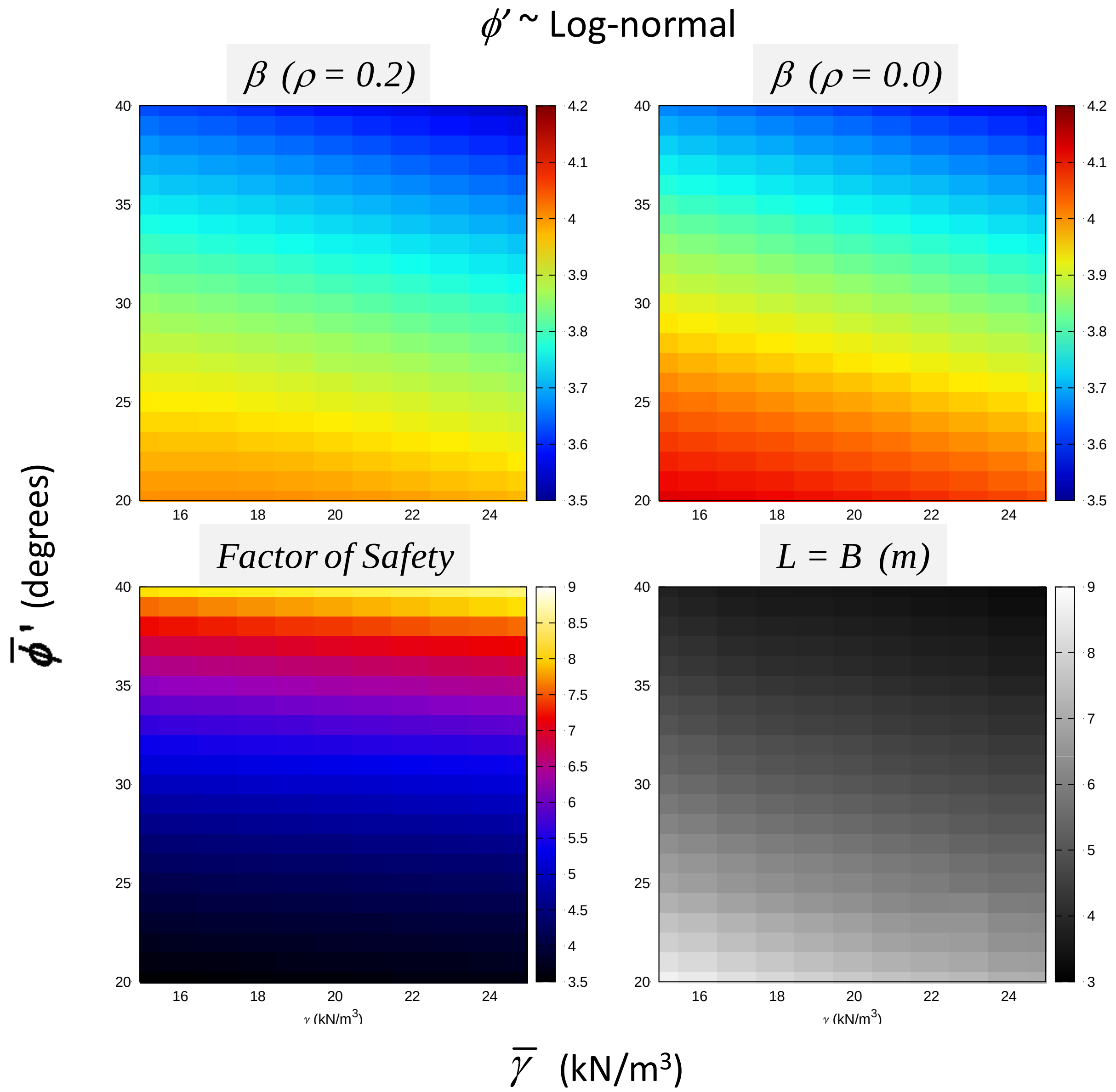


Figure 4: Effect of range of material properties on reliability index and FoS

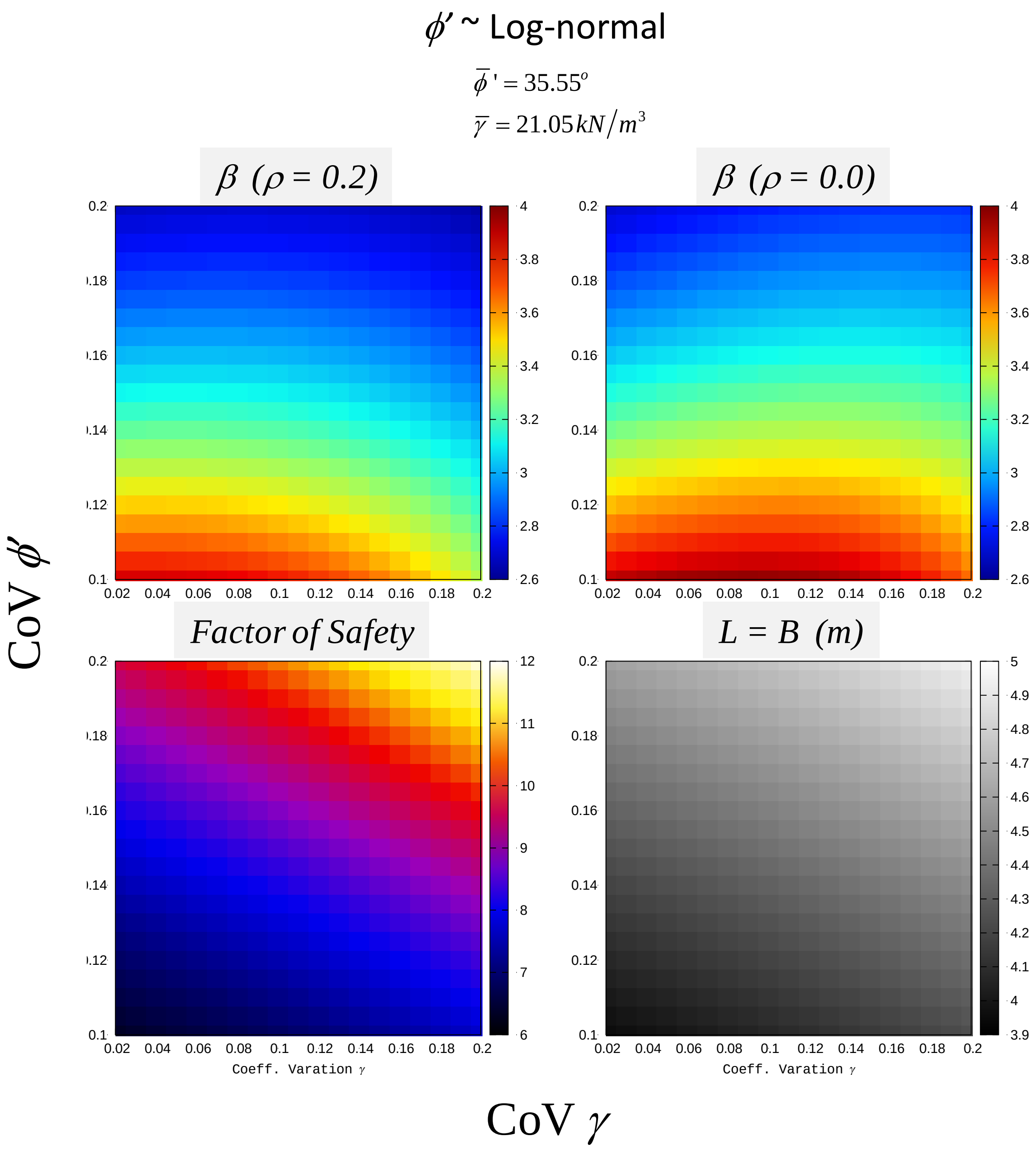


Figure 5: Effect of changes in variance of the soil properties on reliability index and FoS

4. SUMMARY OF RESULTS FOR OTHER EXAMPLES

A similar examination of the reliability of Eurocode-compliant design and the corresponding factors of safety has been carried out for the remaining examples. Only a single set of ground parameters was used. Analyses to assess the effect of the range and variance of ground properties are still underway.

The factor of safety achieved by the Eurocode-compliant design has been calculated using the characteristic and mean values of the loads and material properties.

The results are summarised in Table 3.

Table 3: Reliability indices and factors of safety for remaining examples

Example	No. Variables	Solution B or L	Factor of Safety		Reliability Index β	
			Mean	Characteristic	Monte Carlo	FORM
0	3	3.10m	2.50	5.18	3.45	3.49
1	3	1.97m	2.40	4.86	3.46	3.51
2	4	3.99m	2.60	6.58	3.58	3.69
3	3	8.58m	1.73	2.76	3.35	3.36
4	3	3.52m	3.12	6.88	3.30	3.33
5	3	4.00m	1.63	2.34	3.39	3.40
6	3	2.57m	1.25	1.43	3.23	3.24

5. CONCLUSIONS

5.1 Limit States Design

The starting point for all the above analyses was a EN1997-1 compliant, partial factor limit states design solution to each of the seven common geotechnical problems shown in Figure 1. Four significant conclusions can be drawn from the results.

- The reliability indices are fairly constant for the wide range of problems considered (footings, piles and retaining structures). For the chosen soil properties, the reliability index varied from 3.2 to 3.7.
- Even with a wide range in material properties, the variation in reliability indices for Example 2 was not excessive. For friction angles ranging from 20^o to 40^o and densities from 15kN/m^3 to 25kN/m^3, combined with typical values for the coefficient of variation, the reliability indices varied from 3.5 to 4.2.
- The reliability indices obtained are generally lower than the default target value of β = 3.8 from the Eurocodes, but not significantly so. The manner in which the characteristic value is chosen will affect the level of reliability.
- In all the above analyses, the relationship between the mean value and the characteristic value of the material properties takes account of the expected variance of the parameter. This dependence comes about by selecting the characteristic value to be a multiplier of standard deviations from the mean value with the multiplier being dependent on the degree to which the occurrence of the limit state is affected by the average or the minimum properties of the material. The change in the selected characteristic value due to the change in the coefficient of

variation of the material properties is, however, insufficient to compensate for the effect which higher variance in soil properties has on the reliability index, which is see in Figure 5 to decrease significantly with increases in the variance in soil properties. Caution should be exercised when using limit states design methods in highly variable ground.

5.2 Working Stress Design

Three significant observations are made regarding the factor of safely:

- In contrast to the relatively modest variation in reliability index, the factors of safety obtained from working stress design analyses vary widely for different problem types and across the range of soil properties considered. This supports the now well-established realisation that the factor of safety is a poor means of assessing the reliability of a structure.
- For the range of material properties considered in Example 2, an increase in the factor of safety corresponds with a decrease in the reliability index.
- The factors of safety obtained when the working stress analysis is carried out using characteristic values of loads and material properties are closer to those traditionally used in practice than those obtained using mean values. The factors of safety calculated using the mean values are significantly higher. This supports the views expressed by Krebs Ovesen in the mid-80's that there is not a significant difference between the characteristic values of material properties and the values that would typically be chosen for working stress design methods.

5.3 Reliability Based Design

The analyses performed demonstrate that reliability analysis can be effectively and practically applied to common geotechnical problems. Furthermore, the simplified FORM analyses gave results that compared well with those obtained using Monte Carlo simulation. Spreadsheet applications of FORM (e.g. Low and Tang, 2007) make this a practical tool for use in the design office, particularly for problems with closed form-solutions.

One of the limitations of the reliability analyses described in this discussion note is the assumption that a single value of a material parameter applies at all points in the soil mass and along the full length of any failure surface. Consider, for example, the effect of variations in the friction angle of the soil in the case of the pile foundation in Example 3. The assumption made in both the FORM and Monte Carlo analyses is that the same friction angle applies throughout, i.e. for the calculation of both end bearing and shaft resistance. Thus, although variations in the strength of the ground are taken into account, the spatial variations and the degree to which these variations will be "averaged out" along the length of the failure surface are ignored. In this respect, the approach used in limit states design is more appropriate in that different characteristic values used for end bearing and shaft resistance, taking account of the degree to which the occurrence of the limit state is affected by the average or local properties of the ground.

6. REFERENCES

EN1997-1:2004 Eurocode 7: *Geotechnical design – Part 1: General Rules. European Committee for Standardisation*, Brussels.

Low B.K and Tang W.H. (2007) Efficient spreadsheet algorithm for First-Order Reliability Method. J*. Engineering Mechanics, Volume 133, No 12*. p 1378-1387. ASCE, New York.

Orr T.L.L. (2005) *Proceedings of the International Workshop on the Evaluation of Eurocode 7.* 21 March and 1 April 2005, Trinity College, Dublin.

Phoon K. K. & Kulhawy F. H. (1999) Characterization of geotechnical variability. Canadian Geotechnical Journal 36:612–624.

Retief J.V & Dunaiski P.E. (2009) The limit states basis of structural design for SANS 10160-1. Chapter 1-2. *Background to SANS 10160*. SUN MeDIA, Stellenbosch, S. Africa.

Schneider H.R. (1997) Panel Discussion: Definition and determination of soil properties. 14th International Conference on Soil Mechanics and Geotechnical Engineering, Hamburg, 6-12 Sept. Vol. 4. p2271-2274. Balkema, Rotterdam.