Time-based Auxiliary Bayesian Updating of Embankment Settlement

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Abstract: This study presents an efficient Bayesian framework for real-time predictions of embankment settlement. It consists of two major components, (1) driving Bayesian analysis through BUS in a single simulation run to generate valuable information that will be directly employed in future real-time prediction; (2) target Bayesian analysis to provide posterior settlement predictions, at negligible additional computational efforts, when new monitoring data become available. The proposed approach is illustrated and validated by an embankment settlement prediction example provided by TC304/TC309 for student contest.

Keywords: embankment settlement, Bayesian updating, response conditioning method, BUS

1. Introduction

Bayesian analysis, widely applied for geotechnical performance predictions (e.g., embankment settlement), can be computationally prohibitive for real-time predictions as repeated Bayesian inferences are required due to the timely acquired monitoring data at various monitoring stages. Considering the underlying correlations behind Bayesian analysis when incorporating these data (e.g., $\mathbf{y}^*$ and $\mathbf{y}$), posterior or conditional samples for these data generally overlap. To make use of samples generated when considering $\mathbf{y}^*$ for posterior sampling of $\mathbf{y}$, without repeated Bayesian simulations, this study proposes an efficient time-based auxiliary Bayesian updating framework for real-time predictions of embankment settlement as new data become available. To achieve this, this work starts with the development of the proposed framework, followed by the illustration and validation of the proposed approach using an embankment example provided by TC304/TC309.

2. Time-based auxiliary Bayesian updating

As schematically displayed in Fig. 1, the proposed approach starts with driving Bayesian analysis by selecting driving data $\mathbf{y}^*$ at certain monitoring phase, which can be achieved by BUS with subset simulation (SuS) (Straub and Papaioannou, 2015). During this process, independent subsets (Ω_i, i = 0, 1, …, m) can be obtained based on the m levels of SuS and their probabilities (i.e., $P(\Omega_i)$) can be estimated (Au, 2007). Note that $N+N(1-p_0)(m-1)$ samples are returned, along with settlement evaluations corresponding to each sample in Ω_i, as demonstrated in Fig. 1 through steps (a) to (b).

When monitoring data $\mathbf{y}_j$ (e.g., $\mathbf{y}_j = [y_1, y_2, \ldots, y_j]$) become available, posterior predictions can be performed using response conditioning method (RCM), namely, posterior conditional samples within Ω_i can be directly selected according to the failure event F_j defined as (Straub and Papaioannou, 2015)

$$F_j = \{w - c \cdot L(\boldsymbol{\theta}|\mathbf{y}_j) \le 0\} \tag{1}$$

where w is a standard uniform variable; $\boldsymbol{\theta}$ are a vector of random variables concerned; c is the constant likelihood multiplier, and $L(\boldsymbol{\theta}|\mathbf{y}_j)$ represents the likelihood function reflecting the probabilistic relations of measurements $\mathbf{y}_j$ and predicted quantities $\mathbf{M}(\boldsymbol{\theta})$ through model error ε_j, which is often assumed as normal and additive.

Based on these selected failure samples in Ω_i, shown by filled circles in Fig. 1 (c), another new sample space $O_{j,i}$ is established, and their respective normalized failure probability $P(O_{j,i})$ can thus be estimated. According to BUS, samples in $O_{j,i}$ are distributed as posterior distribution $f(\boldsymbol{\theta}|\mathbf{y}_j)$ with occurrence probability $P(O_{j,i})$, therefore, posterior predictions $\tilde{Y}_t$ at time T_t ($t > j$) can be unbiasedly evaluated as

$$\tilde{Y}_t = \sum_{i=0}^{m} E(M_t(\boldsymbol{\theta})|O_{j,i})P(O_{j,i}) \tag{2}$$

herein, $E(M_t(\boldsymbol{\theta})|O_{j,i})$ denotes the mean predicted quantity conditional on samples in $O_{j,i}$.

Note that $\mathbf{y}_j$ change with time, which indicates that target Bayesian analysis, shown in Fig. 1 (c) and (d), can be repeatedly conducted, where computational efforts remain unchanged, as illustrated in the following section.

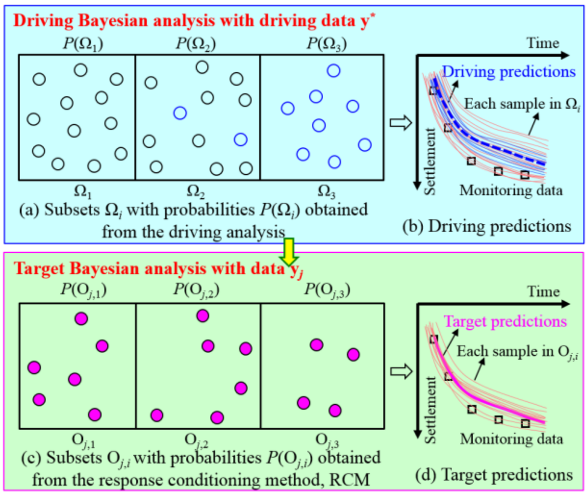

Fig. 1. Schematic diagram of the proposed approach

3. Results and validation

Following the information given by TC304/309 and Jostad et al (2018), numerical finite element model (FEM) of Ballina embankment is firstly established for illustration of the proposed approach. Fig. 2 displays the numerical model, along with the soil layers with their respective constitutive models adopted. Parameters of the estuarine layer are considered probabilistically and their prior statistics are summarized in Table 1, where uniform distributions are assumed. Based on these, the proposed approach can be illustrated with monitoring points (e.g., M0-M3, HPG1 shown in Fig. 2) as follows.

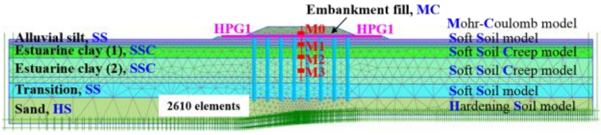

Fig. 2. Finite element model of Ballina embankment

Table 1. Prior statistics of uniform parameters

Parameter	Nominal Value	Typical Range	Nominal Value	Typical Range
	Estuarine clay (1)		Estuarine clay (2)	
λ^*	1.41e-1	[0.84, 1.98]e-1	2.32e-1	[1.39,3.25]e-1
κ^*	2.20e-2	[1.32, 3.08]e-2	3.60e-2	[2.16,5.04]e-2
μ^*	4.25e-3	[1.70, 6.80]e-3	7.00e-3	[2.80,11.20]e-3
φ'	36.0		[20.0, 52.0]	
k_x, m/day	1.00e-3		[0.40, 1.60]e-3	
k_y, m/day	0.40e-3		[0.16, 0.64]e-3	
POP, kPa	24.0		[14.0, 34.0]	

3.1 Results considering M0-M3, respectively

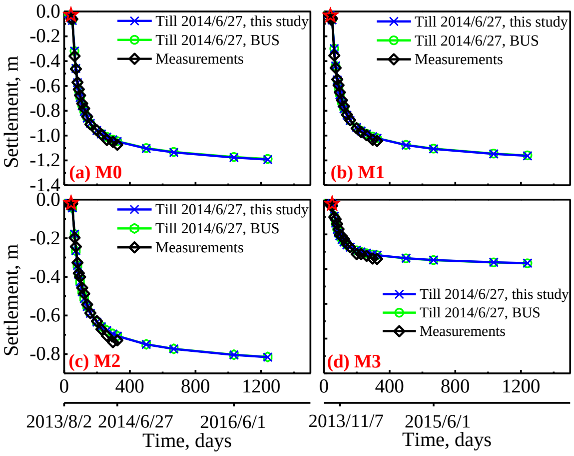

Fig. 3. Settlement predictions considering M0-M3, respectively

Driving Bayesian analysis in this subsection are respectively performed for M0 to M3 by selecting their respective first data as driven data, shown by stars in Fig. 3, from which N = 10000, p_0 = 0.1 for SuS, and $\varepsilon_j \sim N$ (0, 0.1). Then based on these, posterior predictions can be directly estimated as newly required monitoring data become available. For illustration, Fig. 3 displays the predicted settlements considering only one case (i.e., data of 324 days till June 27, 2014), where results from the reference BUS (i.e., N = 10000, p_0 = 0.1) are also plotted for validation. As seen in Fig. 3 (a) for M0, the predicted settlements obtained from the proposed approach, shown by line with "x", have a good agreement with real measurements. Moreover, there is a great consistency between the predicted settlements evaluated from the proposed approach and BUS (shown by line with circles), demonstrating that the proposed method can provide real-time settlement predictions considering different amount of monitoring data. The same observations can be obtained for M1 to M3 as well, shown from Fig. 3 (b) to Fig. 3 (d).

3.2 Results considering measurements in HPG1

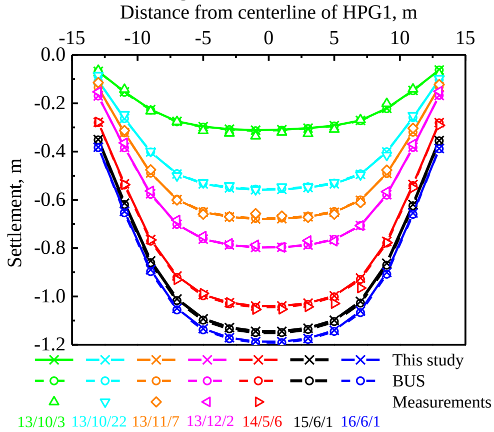

Fig. 4. Settlement predictions considering HPG1

To further demonstrate the proposed approach, this subsection performs driving Bayesian analysis considers 14 monitoring points in HPG1 at Oct. 3, 2013. Similarly, posterior predictions considering the following six phases (i.e., those in Fig. 4) can be sequentially obtained with ease through the proposed approach. For demonstration, settlement predictions incorporating the first five set of monitoring data (e.g., those from Oct. 3, 2013 to May 6, 2014) are displayed in Fig. 4, with each color denoting one monitoring phase. As observed, the proposed method can provide consistent predictions with real data for the five set of data incorporated. Besides, future predictions (e.g., those at June 1, 2015 and June 1, 2016) can be obtained as well. Moreover, the obtained settlements for different monitoring phases are validated with the BUS method and great agreement is achieved. This, once again, demonstrates the proposed approach for real-time Bayesian updating of embankment settlement with more driving data. However, further investigations of the proposed approach are out of the scope of this work.

4. Conclusions

This study proposed an efficient Bayesian framework for real-time predictions of embankment settlement. It starts with driving Bayesian analysis in a single simulation run, based on which target Bayesian analysis is performed for settlement predictions as new monitoring data becomes available. Through an illustrative example, it is demonstrated that the proposed approach can provide satisfactory predictions when compared with real data and is efficient for real-time predictions of embankment settlement.

5. References

Au, S.K., 2007. Augmenting approximate solutions for consistent reliability analysis. *Probabilistic Engineering Mechanics*, 22(1): 77-87.

Jostad, H.P., Palmieri, F., Andresen, L. and Boylan, N., 2018. Numerical prediction and back-calculation of time-dependent behaviour of Ballina test embankment. *Computers and Geotechnics*, 93: 123-132.

Straub, D. and Papaioannou, I., 2015. Bayesian Updating with Structural Reliability Methods. *Journal of Engineering Mechanics*, 141(3): 04014134.