INTRODUCTION

1.1 Stability assessment for rock slopes

The main feature of mechanical behaviour in rock slope instability is that shear takes place along either a discrete sliding surface or within a zone behind the face: if the acting shear force is greater than the shear strength of the rock on this surface, then the slope becomes unstable. The definition of instability will differ depending on the method adopted.

Commonly, the stability of a slope can be expressed in one or more of the following terms (Wylie and Mah, 2004):

1) **Factor of safety**, FS. Stability is quantified by limit equilibrium of the slope: FS>1 means stable slope.
2) **Strain**. Failure is defined by onset of strains great enough to prevent safe operation of the slope, or that the rate of movement exceeds the rate of mining in an open pit.
3) **Probability of failure**. Stability is quantified by probability distribution of difference between resisting and displacing forces (Safety Margin), which are each expressed as probability distributions.
4) **LRFD (load and resistance factor design)**. Stability is defined by the factored resistance being greater than or equal to the sum of the factored loads.

In order to investigate the stability of a rock slope wedge involved in sliding mechanism, the equilibrium method is usually employed and the factor of safety is computed. In the case studied the presence of an anchorage force has been considered (see Fig. 1).

Here, according to Mohr-Coulomb failure criterion for the rock mass behaviour, two different expressions for the factor of safety (FS) have been investigated (De Mello 1988):

\[
FS^+ = \frac{P \cos(\alpha) \tan(\phi) + T \sin(\alpha + \beta) \tan(\phi) + T \cos(\alpha + \beta)}{P \sin(\alpha)}
\]

\[
FS^- = \frac{P \cos(\alpha) \tan(\phi) + T \sin(\alpha + \beta) \tan(\phi)}{P \sin(\alpha) - T \cos(\alpha + \beta)}
\]

where P is the wedge weight; T is the anchor pull; \(\beta\) is the anchor and the slope inclination; \(\alpha\) is the sliding surface inclination and \(\phi\) is the internal friction angle.

Such factor of safety values can be compared with some target values proposed by Terzaghi and Peck (1967) and by Canadian Geotechnical Society (1992) that for earthworks vary between 1.3 and 1.5.
The upper value applies to usual loads and service conditions while the lower value applies to maximum loads and the worst expected geological conditions.

Therefore, apart from the ambiguity in computing procedure, the factor of safety is affected by uncertainties and variability which cannot be avoided.

The study performed below focuses on reliability approach for computing the safety of the slope by accounting for random variables to describe rock resistance parameters.

2 THE CASE STUDIED

2.1 Uncertainties in rock mass characterization

Strength of a rock mass is characterized by strength of intact rock and of discontinuities. Depending on number, orientation and condition of joints the rock mass behaviour is affected by anisotropy and weakness which can lead to the failure when slopes are concerned. Moreover human activity and weathering processes may contribute to increase failure prone conditions by means of increasing acting forces or reducing rock mass resistance.

Besides, the first step in rock slope stability study is the mechanical characterization of rock mass by means of classification systems. Then, after a good analyses of discontinuities and their shear resistance the type of sliding movement can be forecast.

In the proposed case study a rock slope sliding along sliding prone discontinuities is considered (see Fig. 1) where the slope is the rock face one of an open quarry reinforced by an anchorage.

The friction angle of a joint in a rock mass, can be computed as Barton and Choubey (1977):

$$\phi = JRC_n \log_{10}\left(\frac{JCS_n}{\sigma_n}\right) + \phi_r + i$$  \hspace{1cm} (3)

where $\sigma_n$ is the vertical effective stress acting on discontinuity wall; $JRC_n$ is the joint roughness coefficient for joint of actual length; $JCS_n$ is the joint wall compression strength for joint of actual length; $\phi_r$ is the residual friction angle that can be drawn experimentally and “i” is the roughness of discontinuity at large scale.

$JRC_n$ and $JCS_n$ are calculated by Bandis et al.'s formulation (1981), that is:

$$JCS_n = JCS_0 \cdot \left(\frac{L_n}{L_0}\right)^{-0.03/JCS_0}$$  \hspace{1cm} (4)

$$JRC_n = JRC_0 \cdot \left(\frac{L_n}{L_0}\right)^{-0.03/JRC_0}$$  \hspace{1cm} (5)

where $JCS_0$ and $JRC_0$ are joint wall compression strength and joint roughness coefficient respectively computed for reference joint length $L_0$ of 10cm; $L_n$ is the actual joint length. $JCS_0$ can be provided by reference tables whereas $JRC_0$ can be drawn from Schmidt’ hammer test.

As explained before, the evaluation of $\phi$ for a joint in a rock mass is affected by variability and uncertainty. Here, the friction angle probability distribution is modelled as lognormal with the coefficient of variation equal to 30, 40 and 50%.

Nonetheless, the sliding plane inclination, named $\alpha$, can be considered also as a random variable. It is determined by means of structural investigations and consequently affected by human errors. Therefore, for $\alpha$ a normal probability distribution with a mean value of 40° is considered a coefficient of variation equal to 10%.

2.2 Factor of safety method

The commonest approach to stability studies is based on the computation of the factor of safety. According to the equilibrium method and Mohr-Coulomb soil behaviour, the safety factor is computed by means of different expressions, namely Eq. (1) and (2): such two cases arise from different interpretation of the pull component along the sliding surface direction. As a matter of fact, it can be taken as a stabilizing force or as a negative contribution to the sliding weight component.

The choice between the two approaches can be rationally undertaken considering a parametric deterministic and probabilistic study where anchorage pull and the height of the slope are varied.

Table 1 shows values of variables used in such a benchmark.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean value</th>
<th>Standard deviation value</th>
<th>Characteristic value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sliding surface slope $\alpha$ [$^\circ$]</td>
<td>40</td>
<td>4</td>
<td>38</td>
</tr>
<tr>
<td>Rock face slope $\beta$ [$^\circ$]</td>
<td>7.5</td>
<td>-</td>
<td>7.5</td>
</tr>
<tr>
<td>Friction angle $\phi$ [$^\circ$]</td>
<td>35</td>
<td>10.5</td>
<td>29.8</td>
</tr>
<tr>
<td>Unit weight $\gamma$ [kN/m$^3$]</td>
<td>23</td>
<td>0.5</td>
<td>22.8</td>
</tr>
<tr>
<td>Slope height $H$ [m]</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

Three values for slope height have been investigated: 5, 10 and 15m. According to anchor pull increasing the two factors of safety increase with a different trend as can be seen in Fig. 2.

The expression Eq. (1), indicated as $FS^+$ increases linearly although slope decreases when height increases. So that, the factor of safety is stronger affected by anchorage pull for smaller height of the slope.
On the contrary, expression Eq. (2), reported as FS, shows a parabolic trend with a rapid increase as anchor pull increases.

![Figure 2. Variation of safety factor (Eq. 1) with the anchor pull.](image)

Such a trend provides obviously negative values as the contribution of anchor age pull got higher than the weight component on the weak plane.

Accordingly, FS won’t be discussed further on.

### 2.3 Reliability method

As the FS fails, only expression Eq. (1) can be considered for the safety factor. Then, it is worthy to compare the safety factor with the reliability index computed by means of a random variable approach. Such a parameter is strictly related to the probability of failure. Many studies have been performed on the allowable values of the probability of failure of the engineering project accepted by people. In the case of slope stability values of reliability index about 2-4 are suggested by Baecher (1982).

Accordingly, in this analysis such interval will be investigated.

Table 2. Random variable distribution type.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Variation</th>
<th>Min value</th>
<th>Max value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sliding surface slope $\alpha$ $[, ^{\circ}]$</td>
<td>Normal</td>
<td>10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rock face slope $\beta$ $[, ^{\circ}]$</td>
<td>Uniform</td>
<td>-</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Friction angle $\phi$ $[, ^{\circ}]$</td>
<td>Lognormal</td>
<td>30, 40, 50%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit weight $\gamma$ $[kN/m^3]$</td>
<td>Normal</td>
<td>2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anchorage pull $T$ $[kN]$</td>
<td>Constant</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope height $H$ $[m]$</td>
<td>Constant</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 summarizes mean and standard deviation values for random variables while Table 2 shows the assumption about probability distribution for such provided variables.

As the unit weight $\gamma$ is concerned, it is can be assumed as a random variable with a small variability, proved by literature (Cherubini 1997). In this case it is considered normally distributed with a coefficient of variation equal to 2%.

In this benchmark FORM, SORM and Monte Carlo methods are performed in order to evaluate the reliability index by means of COMREL code (1997). In this case, the performance function considered is:

$$P \cos(\alpha) \tan(\phi) + T \sin(\alpha + \beta) \tan(\phi) + T \cos(\alpha + \beta) - P \sin(\alpha) > 0$$

(6)

Results from FORM and SORM techniques coincide, thus just FORM are reported in the following.

At first, reliability index is reported versus the anchorage pull, as already done for the Factor of safety (Fig. 2). Then, reliability index is compared with the safety factor for the same anchorage pull values.

Fig. 3 shows curved trends of reliability index versus anchorage pull. For the case of slope height equal to 5m, as anchorage pull varies from 100 to 150 kPa the reliability index increases from 2 to 4 when the friction angle coefficient of variation CV$\phi$ is 30%. Such trend can be seen for other two height values but with a slope reduction as the height increases: this means that when height increases an higher anchorage pull increase is needed for stabilizing the slope as in the case of the factor of safety.

![Figure 3. Reliability index versus anchorage pull for CV$\phi$=30%](image)

![Figure 4. Reliability index versus anchorage pull for CV$\phi$=40%](image)

Such evidence is true also when CV$\phi$ increases.
the safety level of the slope reduces and much higher anchorage pull increments are needed for improving stability condition.

Therefore, the reference values 1.3-1.5 suggested for the factor of safety are definitively inadequate when \( \phi \) variability is taken into account.

As the other two slope height cases (H=10m and 15m) are concerned they give the same results as coincident curves are provided.

Results from MCS with an adaptive sampling scheme have been investigated: 30000 samples are needed for the case studied and reliability index values drawn are the same as FORM. Fig. 7 shows the same evolution of reliability index curves from MCS.

2.4 **LRFD method**

Finally, an interesting comparison is proposed between reliability index and the partial factor design method suggested by Eurocode 7 (1997) and by Italian Technical Building Code (TU 14/01/2008).

The combination of factors suggested for the global stability analysis is named as Combination 2, where load factors (A2), resistance factor (M2) and global factor (R2) are considered as follows:

\[
\left( P \times 1.0 \times \cos(\alpha) \tan(\phi) / 1.25 + \\
T \sin(\alpha + \beta) \tan(\phi) / 1.25 + T \cos(\alpha + \beta) \right) / 1.1 > 0 \quad (7)
\]

In Eq. (7), design variables values employed are characteristic values: such values are introduced by the Eurocode for limit state design but no suggestions are provided about the way to compute them.

Characteristic values for reinforced concrete compression strength is commonly assumed as the value corresponding to the 95% probability to be exceeded. Assuming it is normally distributed we have:

\[
x_c = x_m - 1.96 \times s \quad (8)
\]

where \( x_m \) is the mean value and \( s \) is the standard deviation of the concrete compression strength. Such an assumption could be too much conservative in the case of geotechnical random variables. As a matter of fact, let’s consider the friction angle, with mean value equal to 35° and standard deviation equal to 10.5°; its characteristic value is 14.4°, which is too much conservative value.

Here the expression from Schneider (1997) reported also by Cherubini and Orr(1999) is used:

\[
x_c = x_m \left(1 - CV \right) / 2 \quad (9)
\]

where \( CV \) is the coefficient of variation; \( x_m \) is the random variable mean value. Three cases are then considered according to the \( CV_\phi \) variation. In this case, the friction angle characteristic value is 30°.
Moreover, as Eq. (7) is concerned, building code requests that such difference is higher than zero although no minimum value suggests. Hence, Figs. 8-10 show the positive difference between resistance and action according to the expression Eq. (7) versus the anchorage pull values. The trend, for each slope height, is not linear and it shows a decrease of slope as height increases.

The three values of CVφ do not affect the slopes of the trend while affect the magnitude of anchorage pull needed: when CVφ increases the anchorage pull must be increased in order to have the same positive difference between resistance and action.

A meaningful correlation can be observed in Fig. 11 where reliability index is related to the positive difference between resistance and action for the same anchorage pull value. Each graph corresponds to different CVφ values for the case of 5m slope height.

From Fig. 11 the LRFD differences between 30 and 60kPa correspond to the reliability index varying between 2 and 4 for the case of CVφ=30%.

Such difference shall be the same for higher variability although the lower boundary of the range considered increases as friction angle variability increases. This means that, in such a case, simply considering:

1) resistance just higher than actions;
2) characteristic values and partial safety factors; it cannot give a constant reliability level. These changes are related both to anchorage pull values, slope height and variability.

Hence, when variability of the friction angle is enlarged higher differences between resistance and loads are needed but these values vary according not only to CVφ but also to slope height variation.

3 CONCLUDING REMARKS

A benchmark has been proposed to investigate stability of an anchored rock slope according to three different methods: the factor of safety, the reliability index and the load and resistance partial factor.

Results show that, when variability of those random variables which govern the limit equilibrium of a rock slope has taken into account:
1 it causes a safety reduction which depends on variability magnitude;
2 target values for safety factor shall be increased according to variability magnitude;
3 the difference between factorized resistance and load must be increased according to the variability magnitude and the height of the slope.

REFERENCES


