Time capsule for geotechnical risk and reliability

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Abstract

This paper is motivated by the time capsule project (TCP) of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The historical developments of geotechnical risk and reliability primarily for soil mechanics was covered over the past six or more decades (1960 – 2010+). The key features distinguishing geotechnical and structural engineering are the natural origin of the ground and the lack of sufficient data to characterize the ground using the more familiar frequentist interpretation of probability. For the first feature, random field theory is applied to model spatial variability and the random finite element method is applied to solve soil-structure interaction problems in spatially variable soils. For the second feature, compilation of databases is essential to serve as priors for Bayesian updating and the more recent research in Bayesian machine learning. There is a gradual evolution towards reliability-based design as a result of this body of research. Interest in both research and practice has heightened significantly, because probabilistic methods offer a pathway to address big data and implement data-centric geotechnics as one step towards digital transformation. Given the complexity of the natural ground (known unknowns can be large and there are unknown unknowns), engineering judgment remains important to bridge the gap between theory and reality. However, the role of engineering judgment needs to be updated as modern machine learning methods become more powerful.

Keywords: time capsule; spatial variability; geotechnical reliability; databases; machine learning

Introduction

Geotechnical engineering can be distinguished from other civil engineering branches by the need to grapple with very large uncertainties in material parameters and possibly unknown unknowns (e.g., geologic "surprises") (Phoon 2017). For geotechnical engineers, the most important material is natural soil. Therefore, they must constantly struggle with the spatial variability of the substrate, the uncertainty of the stratigraphy, the possibility of the presence of weak soil lenses, and other geotechnical-related uncertainties. Uncertainty quantification in geotechnical engineering is an interdisciplinary issue. It combines, for instance, probability theory (or other uncertainty quantification theory), soil mechanics, natural hazards, geology, and social issues. It is now generally accepted to perform the risk analysis associated with the subsoil for each significant structure. Some past failures clearly show the need for geotechnical engineers to manage uncertainty more rationally. Subsoil-related uncertainties motivated engineers and scientists to describe these effects by using probabilistic

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approaches and incorporating them into existing deterministic models or creating new approaches. These efforts are briefly described below in a short history of uncertainty quantification in geotechnical engineering. A historical review covering so many diverse topics within a single paper is necessarily incomplete and not representative of the volume of important research conducted over the past 6 or more decades (1960 – 2010+). The spirit of this paper is to take the reader on a journey akin to a wine tasting tour, where the sampled wines are the ideas that have contributed to education, research, and practice. In addition, there is a large literature on risk and reliability in rock mechanics, rock engineering, tunnelling, mining engineering, and hydrogeology that is not covered in this tour. Some later examples include Einstein (1991), Priest and Hudson (1981), Hoek (1999), Contreras and Brown (2018), Kitanidis (1997), and Brown (2012), but earlier papers on statistics and reliability have been published in the US and international rock mechanics conferences in the 70s. The application of risk-based design of high rock slopes in South African and Australian mining engineering activities is also noteworthy. Einstein (2003) reviewed how uncertainty has been dealt with over the past 40 years in rock mechanics using the decision making cycle as a frame of reference. Hadjigeorgiou (2019) discussed how different risk analysis tools and procedures can be used effectively in geomechanical mining. Finally, structural reliability is a more mature field with origins traced to Freudenthal (1947) and Pugsley (1955). It is outside the scope of this review as well.

In the geotechnics community, interests in probabilistic and related risk topics likely emerged in the sixties. During the Terzaghi Lecture in 1964, Casagrande (1965) presented the study titled: "Role of the 'Calculated Risk' in Earthwork and Foundation Engineering" where the need for risk assessment in geotechnics was indicated. Shortly thereafter, Lumb's classical Canadian Geotechnical Journal paper on "The Variability of Natural Soils" was published and became one of the first to characterize spatial variability statistically (Lumb 1966; Phoon 2020). Wu is also regarded as one of the seminal figures who contributed to the development of geotechnical reliability (Baecher and Christian 2019). In 1965, Hansen (1956,1965) proposed the selection of partial factors based on two guidelines: (a) a larger partial factor should be assigned to a more uncertain quantity, and (b) the partial factors should result in approximately the same design dimensions as that obtained from traditional practice. It should be noted that the partial factor is applied as a divisor on the nominal soil parameter, probably a matter of convention established by the factor of safety. For the load and resistance factor design (LRFD) format adopted in North America, the resistance factor is a multiplier. Although Hansen's guidelines were qualitative, this is the first time a link between uncertainty in the material parameter and a partial factor that influences design was proposed. Brinch Hansen was the first to use "limit state design" in geotechnical engineering. The term "limit state design" refers to a design philosophy or a design method for ensuring safety. This limit state design method can be non-probabilistic (partial factors calibrated to produce designs comparable to those produced by the global factor of safety) or probabilistic (simplified reliability-based design) (Phoon et al. 2003a). Christian and Baecher (2015) pointed out that Taylor (1948) had discussed the use of partial factors for cohesion and friction angle to account for the different uncertainties underlying the estimation of these parameters, although load partial factors and limit states were not considered. There are also other scholars that laid the foundation for different aspects of geotechnical risk and reliability in the sixties as outlined in the next section.

Casagrande (1965) defined the term "calculated risk" as:

"*a) The use of imperfect knowledge, guided by judgment and experience, to estimate the probable ranges for all pertinent quantities that enter into the solution of the problem.*

b) The decision on an appropriate margin of safety, or degree of risk, taking into consideration economic factors and the magnitude of losses that would result from failure."

He also introduced the terms "unknown risk" and "human risk" and reviewed several case histories. Casagrande tried to demonstrate the importance of risks in earthwork and foundation engineering. Although the paper by Casagrande served as a turning point in risk assessment in geotechnics, some papers dealing with this subject were written in non-English languages. Among them, we can distinguish four studies related to the safety of geotechnical structures: Wastlung (1940) written in Swedish, Levi (1958) written in French, Biernatowski (1966a,b) written in Polish,and Hansen (1956) written in Danish.

The development of uncertainty quantification in geotechnical engineering is a continuous process, but sometimes it is drastically accelerated by innovative approaches and new techniques. The work of Casagrande, Hansen, Lumb, Wu, and others in the sixties can be considered as an approximate origin where the discipline of uncertainty quantification in geotechnical engineering began to take shape. The expansion of this discipline due to this continuous process cannot be divided into separate periods; however, to bring some structure to our story, we adopted the convention of distinguishing each decade, and our effort was focused on identifying the most important achievements and ideas in those decades. A rather illuminating example of how the discipline developed is shown in Fig. 1, where the number of published papers mentioning the given words in Google Scholar is compared. Fig. 1a shows a growing interest in geotechnical-related uncertainty quantification, which is a response to the infrastructure needs of the modern world. Moreover, an increasing proportion of studies related to uncertainty issues to general geotechnical studies is observed, as shown in Fig. 1b.

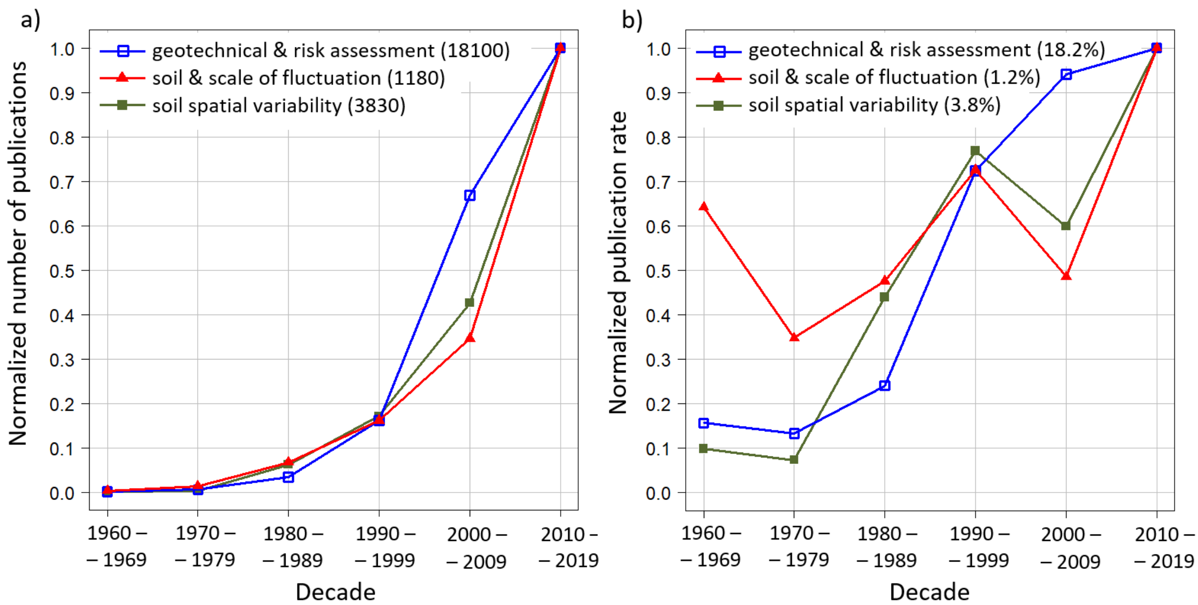

Fig. 1. Normalized number of publications containing specific phrases; (a) normalization is with respect to total number in the 2010-2019 decade (shown in parentheses); (b) normalization using the ratio of the number of publications containing specific phrase to the number of publications containing the phrase "geotechnical engineering" (percentage in the parenthesis refers to ratio in the 2010-2019 decade).

Returning to our story, we start with the first steps taken by geotechnical engineers to grapple with diverse and significant uncertainties pertaining to the subsurface environment with focus on soils. The description of the pioneering decade begins.

1960-1970

The study that deserves to be mentioned first is the pioneering Lumb (1966) paper. He titled his study "The variability of natural soils" and it is considered to be the starting point for a new branch of geotechnics (Phoon 2020). Lumb showed that the variations in properties for natural soils can be described by random variations about a mean or linear trend, related to a probability distribution. As a basis, he considered four typical Hong Kong soils – a soft marine clay, an alluvial sandy clay, residual silty sand, and residual clayey silt. The studied properties include Atterberg limits, grading, and, for undisturbed samples, strength and compressibility characteristics. He provided examples of soil properties following the normal, lognormal, and binormal distributions. A relatively large sample size was an advantage of Lumb's study. Moreover, Lumb stated that the probability the parameter could be less than the design value is a rational basis for the choice of design parameters. It was the first study to guide later research on application of probability to quantify geotechnical parametric uncertainties numerically. For example, Eurocode 7 (Comité Européen de Normalisation 2004), Section 2.4.5.2 Clause 11 states "If statistical methods are used, the characteristic value should be derived such that the calculated probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5%." This fractile definition of the characteristic value can be found in Lumb (1966). Details are given elsewhere (Orr 2015). At the same time as Lumb's classical paper was published, other studies concerning the application of probability theory to engineering structures began to appear. The paper by Langejan (1965) discussed the slope stability problem. Biernatowski (1968) also considered probabilistic solutions for slope stability. One year earlier a pioneering paper by Wu and Kraft (1967) was published. The authors used the probability of failure to quantify the safety of foundations for several load and strength distributions. They determined the appropriate probability function (a normal distribution was used) by fitting the experimental data. Kraft's dissertation (Kraft 1968) and related papers (Wu and Kraft 1967, 1970a, 1970b) were some of the first probabilistic assessments of foundation safety for bearing capacity and settlement. Wu (1974) summarized and extended this body of work in his 1974 paper on "Uncertainty, Safety, and Decision in Soil Engineering". In principle, measurement error can be determined directly by analyzing the variation of the results obtained by a representative group of soil testing companies performing the "same" test on nominally identical soil samples. Comparative testing programs of this type were conducted by Hammitt (1966) and Johnston (1969). The above-described studies laid the foundations for the wider use of probabilistic methods to describe uncertainties in geotechnics, which was revealed in the next decade.

1970-1980

The initial steps in describing uncertainties in geotechnical engineering in the sixties ushered in a period of tremendous progress in the seventies. There are a few reasons for this. One of them is Lumb's initiation of a conference series dedicated to probability theory applications in geotechnics, i.e., the International Conference on Applications of Statistics and Probability to Soil and Structural Engineering (ICASP). This conference series promotes a probabilistic viewpoint on soil parameters among other topics. In the 1970s, three ICASP conferences took place, Hong Kong in 1971, Aachen in 1975, and Sydney in 1979. Another reason for the increasing popularity of the probabilistic approach for uncertainty description is the development of appropriate computational methods, e.g., calculation of the first-order reliability index by Hasofer and Lind (1974) and an algorithm for the calculation of structural reliability under combined loadings by Rackwitz and Flessler (1978). In the seventies, Lumb continued to investigate probabilistic descriptions of soil parameters, e.g., in his 1970 paper (Lumb 1970), he compared the natural variabilities of cohesive and frictional components of strength and found that the beta probability distribution agrees more closely with experimental data than the commonly assumed normal distribution. Lumb also authored 'Chapter 3: Application of Statistics in Soil Mechanics' in the book "Soil Mechanics – New Horizons" (Lumb 1974). After Lumb, studies by other researchers aiming at a probabilistic description of subsoil parameters started to appear. Soil parameters as random variables were investigated by Schultze (1972, 1975). An extensive study for multiple samples was conducted by Corotis et al. (1975). The authors showed the results in three groups based on the bulk density of soil. They found that the probability characteristics depend on the bulk density of the soil and therefore on its type and that most of the variations can be described by either normal or lognormal distribution. The statistical characterization of soil properties remained an important topic of interest (e.g., Singh and Lee 1970; Kay and Krizek 1971; Haldar and Tang 1979a). Apart from describing uncertainties in soil properties, more applications to geotechnical structures became accessible in the seventies. Among important studies of that time, one can find a study by Singh (1972), where the investigation on how reliable is the factor of safety in foundation engineering is performed. The author investigated retaining structures, bearing capacity problems, and slope stability analyses. The stability of rigid structures in the probabilistic formulation was also considered by Biernatowski (1972). In 1976, a classic paper on slope stability analysis was published by Alonso (1976). Alonso tried to define better slope safety measures by using a probabilistic approach. He used a mechanistic description of stability (the method of slices) for Canadian sensitive clays and implemented first-order probability analysis to allow rational evaluation of the different sources of uncertainty. He found that the uncertainties in the cohesion parameter, the pore-pressure, and the method of analysis are the relevant ones governing the uncertainty in the estimation of slope safety. Probability-based short-term and long-term designs of soil slope were investigated by Tang et al. (1976) and Yuceman and Tang (1975), respectively. Wilson H. Tang contributed significantly to the reliability analysis of slopes, design of offshore foundations, and the promotion of the use of Bayesian methods in geotechnical engineering (Lacasse et al. 2017). His lifetime contribution to the field of geotechnical reliability was summarized in ASCE GSP 286 (Juang et al. 2017) and Proceedings of the Professor Wilson Tang Memorial Symposium (Zhang et al. 2012). de Mello (1977) is the first Rankine lecturer to discuss the application of statistics and probability to decision making in dam engineering. He expanded on the central idea in his General Report at the second ICASP in Aachen 1975 that the statistics of averages is distinct from the statistics of extreme values (de Mello 1975). This distinction is now codified in Eurocode 7 (Comité Européen de Normalisation 2004), Section 2.4.5.2 "Characteristic values of geotechnical parameters", Clauses 7 and 8. Clause 7 states that the "zone of ground governing the behaviour of a geotechnical structure at a limit state is usually much larger than a test sample or the zone of ground affected in an in situ test. Consequently the value of the governing parameter is often the mean of a range of values covering a large surface or volume of the ground. The characteristic value should be a cautious estimate of this mean value." The statistics of averages is thought to apply to this "large surface or volume of the ground." A specific example of slope stability was highlighted by de Mello (1977). However, it is now understood that the statistics of averages and the statistics of extreme values are both needed in the presence of spatially variable soils, because critical slip curves seek the weakest kinematically admissible paths by definition (Hicks and Samy 2002; Ching and Phoon 2013; Tabarroki et al. 2020).

The statistics of spatial averages in spatially variable soils was first studied formally by Vanmarcke using random field theory (Vanmarcke 1977a, 1977b). Vanmarcke stated that when the degree of disorder is sufficiently large, there is usually merit and economy in probabilistic rather than deterministic models. He pointed out that random field theory seeks to model complex patterns of variation (interdependence, correlation) in cases where deterministic treatment is inefficient and conventional statistics are insufficient. Vanmarcke stated that central to the development of robust random field models is the concept of the “local average” of a random field that is more relevant to geotechnical engineering practice than point variation. Vanmarcke’s “local average” is different from the “statistics of averages” of de Mello (1977) in one key aspect. He considered spatial variability or spatially correlated soil properties, which is visibly present in all soil profiles. Vanmarcke further stated that the ideal random field model should capture the essential features of a complex random phenomenon by a minimal number of physically meaningful and accessible parameters. His studies began a new branch of soil parameters description by using random fields and were widely used in the following decades.

The probability approach has been also incorporated into dynamics issues, e.g., Haldar and Tang (1979b) proposed a procedure to estimate the probability of liquefaction for a given design earthquake magnitude and acceleration, initiating the research on the application of probabilistic methods in soil liquefaction problems. At the end of the 1970s, the first geotechnical applications of the stochastic finite element method (FEM) appeared, e.g., stochastic analysis of steady-state groundwater flow in a bounded domain by Smith and Freeze (1979a, b). However, the finite element-based approaches found wider use in the next decade. The increasing power and decreasing cost of computing machines should explain this rising popularity in later years. Phoon et al. (2022a) opined that the increasing power and convergence of digital technologies beyond computing machines will usher the next wave of change in research and practice.

The amount of uncertainty involved in a geotechnical design can be reduced through additional information. Such an uncertainty reduction process can be formally formulated based on Bayes’ theorem. In the 1970s, the potential of uncertainty updating through Bayes’ theorem had been recognized by Wilson H. Tang. It is worth highlighting that Tang (1971) introduced how to quantify the value of additional data in geotechnical engineering through Bayesian analysis. Tang (1973) systematically illustrated the concept of how the Bayesian method can potentially be used for uncertainty reduction in geotechnical engineering. Kay (1976) suggested a Bayesian method to reduce the uncertainty associated with the bearing capacity of pile foundations through load test data, which can then be used to improve pile design using reliability theory.

In 1970, Benjamin and Cornell (1970) published the first textbook which systematically introduces the knowledge of probability and statistics for students, practitioners, teachers, and researchers in civil engineering, and was inspiring for early researchers in geotechnical engineering. The classic textbooks by Ang and Tang (1975, 1985), which provided comprehensive examples on how probabilistic methods can help improve civil engineering analysis, made the concepts and methods of reliability analysis much more accessible to the geotechnical profession. These textbooks were revised in 2007 (Ang and Tang 2007), and have been translated into 5 languages and used worldwide.

As early as the 1970s, researchers at the Norwegian Geotechnical Institute (NGI) started to contribute to geotechnical risk and reliability. Folayan et al. (1970) developed a Bayesian updating method to predict the settlements of a marshland development and analyzed the associated economic consequences. Høeg and Murarka (1974) adopted a probabilistic approach to calculate the probability of failure of a retaining wall. A number of these ideas originated from the PhD studies of Folayan and Murarka was conducted at Stanford University.

This is also a period of overlap between geotechnical reliability and the more mature field of structural reliability (Cornell, Veneziano, Rackwitz, Madsen, Schuëller, Corotis, Hasofer and Lind, Ditlevsen, Thoft-Christensen, Melchers, etc.). The literature on structural reliability is extensive. For the past five decades, twelve International Conference on Structural Safety and Reliability (ICOSSAR) have been organized by the International Association for Structural Safety and Reliability (IASSAR). The first ICOSSAR was held in 1969 in Washington, D.C., USA. Since 1977, it has been successfully held every four years at venues in Europe, USA and Japan: Washington, D.C., USA (1969), Munich, Germany (1977), Trondheim, Norway (1981), Kobe, Japan (1985), San Francisco, USA (1989), Innsbruck, Austria (1993), Kyoto, Japan (1997), Newport Beach, USA (2001), Rome, Italy (2005), Osaka, Japan (2009), New York, USA (2013), Vienna, Austria (2017) and Shanghai, China (2022).

1980-1990

The growing popularity of probabilistic methods in geotechnics enhanced by new computational methods, e.g., Ditlevsen (1981a) and Hohenbichler and Rackwitz (1981), resulted in increased activity by researchers in this field. There were two ICASP conferences in the 1980s decade, Florence in 1983 and Vancouver in 1987. The ASCE proceedings “Probabilistic Characterization of Soil Properties: Bridge between Theory and Practice” published in 1984 is noteworthy (Bowles and Ko 1984). In 1983, a classic book by Vanmarcke was published, “Random Fields – Analysis and Synthesis” (Vanmarcke 1983). Tang (1984) proposed a simplified method to estimate spatially-averaged soil properties. In 1983, McAnally (1983) investigated the reliability of the bearing capacity designs of shallow footings in sands and proposed a procedure that allows foundations to be designed with a more consistent level of reliability than those obtained by using a single value of safety factor. Biernatowski and Puła (1988) proposed an approach for probabilistic analysis of the stability of massive bridge abutments. In 1988, an original approach of Kandaurov's stochastic model of granular soil was developed by Brząkała (1988).

However, the greatest interest was in the slope stability analysis, e.g., Li and Lumb (1987) discussed some improvements on the first-order second-moment (FOSM) probabilistic approach to slope design, which had originally been introduced by Cornell (1971) at the first ICASP conference in Hong Kong. Ditlevsen (1981b) extended Cornell’s FOSM to large systems, which is important for real-world problems. Chowdhury et al. (1987) investigated the progressive development of slope failure within a probabilistic framework. Ishii and Suzuki (1986) proposed the stochastic finite element method (FEM) that uses the first-order approximation at a failure point of a set of random variables. Baecher et al. (1980) conducted one of the first studies on the risk assessment of dams, and a more comprehensive treatment was presented by Hartford and Baecher (2004). A state-of-the-art for dam safety risk analysis was reviewed by Baecher (2016).

The development of probabilistic methods in geotechnics was also motivated at that time by the need of designing drilling platforms, offshore structures, and their foundations as experiences on the analysis, design, and construction of such structures are rare. Wu et al. (1989) provided a state of the art review on the reliability of offshore foundations. The need for proper soil spatial variability characterization became important for marine soils (e.g., Wu et al. 1987). Kraft joined the offshore oil business after his PhD at the Ohio State University and continued to publish seminal papers in this field (e.g., Kraft and Murff 1976). Initiated by these pioneering studies, offshore foundations have become one of the areas where geotechnical reliability method has found successful applications.

Robert V. Whitman in his Terzaghi lecture in 1981 opined that “probability theory is regarded with doubt and even suspicion by the majority of geotechnical engineers” (Whitman 1984), and he mentioned the language barrier as one of the possible reasons for this situation. However, fortunately, this decade can be considered as a turning point in the attitude of geotechnical engineers to probabilistic methods. This is mainly due to good examples of how these methods have been applied in practice. Important studies that investigated real geotechnical structures from a probabilistic point of view were published in the 1980s. Among them was a study by Duncan and Huston (1983) on estimating the probability of failure of California Delta levees by using simple statistical procedures. Their analyses were based heavily on empirical data. An interesting study of probabilistic analysis in the assessment of dam safety issues was published by Vick and Bromwell (1989). Their examples showed that probabilistic methods were expanding into geotechnical engineering practice. It is worthwhile pointing out Baecher's contribution to making geotechnical risk and reliability more accessible to non-specialists. Examples include the FHWA "Geotechnical Risk Analysis User's Guide" (Baecher 1987), and two special MIT summer courses, "Geotechnical Error Analysis" (Baecher 1985) and "Reliability Analysis of Stability of Embankments of Soft Clays" (Baecher and Ladd 1985).

The 1980s also saw a significant development in the applications of the stochastic FEM in geotechnics. Baecher and Ingra (1981) used the stochastic FEM to predict uncertainties in total and differential settlements under a large flexible footing. Righetti and Harrop-Williams (1988) modelled a structure by a limited number of accessible stochastic data and computed the characteristics of the displacement and stress random fields in the structure by the first-order second-moment approximation. The authors considered the application of stochastic FEM for a soil profile with a random distribution of the elastic modulus.

In the 1980s, the benefits of Bayesian methods in geotechnical engineering were further explored. Notably, Baecher and Rackwitz (1982) suggested that the variability of the bearing capacity can be divided into within-site and cross-site variabilities and illustrated how the cross-site variability can be reduced through the Bayesian method. The above concept later initiated the development and application of hierarchical Bayesian models for geotechnical engineering applications in the 2020s (e.g., Zhang et al. 2014, 2016; Bozorgzadeh and Bathurst 2020; Ching et al. 2021a; Xiao et al. 2021). Tang and his co-works developed Bayesian methods for detecting anomalies through site investigation (e.g., Tang and Quek 1986; Tang 1987; Tang et al. 1988; Tang and Halim 1988).

As one can observe, the 1980s brought tremendous development of probabilistic approaches in geotechnical engineering. For completeness, it is worthwhile to point out the work of Harr (1977, 1987), which is somewhat outside the mainstream development but nonetheless influential on the U.S. Army Corps of Engineers through the students he supervised at Purdue University and who later went on to work there, especially Wolff (1985, 1994, 1995). A more complete summary of this body of work is given elsewhere (Wolff 2008). As a derivative of this, the next decade brought a wider application of probabilistic methods to geotechnical engineering.

1990-2000

An important aspect influencing the development of probabilistic methods in geotechnics in the 1990s was the growing computing power of computers (especially personal computers). This made it possible, for example, to use a combination of the Monte Carlo method and the finite element method (named random FEM, or RFEM). Such an approach was intensively developed after the publication of the first studies by Fenton and Griffiths. They investigated seepage beneath water retaining structures founded on spatially random soil (Griffiths and Fenton 1993) and estimated the distribution of an equivalent conductivity measure, the block conductivity, which characterizes the total flow rate through a two-dimensional bounded domain and which is itself a random variable (Fenton and Griffiths 1993). In the paper by Paice et al. (1996), RFEM was used for settlement modeling on spatially random soil. Stochastic FEM was further developed in the 1990s (e.g., Spanos and Ghanem 1989) and used for other geotechnical applications, e.g., analysis of soil layers with random interfaces (Ghanem and Brząkała 1996). Simpler perturbation-based stochastic FEM was explored by Phoon et al. (1990) and Quek et al. (1991, 1992). Foundation settlements for layered soil were also studied using stochastic FEM by Brząkała and Puła (1996). In the 1990s, ICASP conferences continued and provided a platform for the exchange of ideas in the geotechnical research community, in this decade three ICASP conferences took place, i.e., Mexico City in 1991, Paris in 1995, and Sydney in 1999.

The increasing interest in using the random field for soil spatial variability description resulted in a growing need for random field properties estimation and simulation. In response many important studies on soil parameter variability were published in the 1990s, e.g., Fenton and Vanmarcke (1990), Kulhawy et al. (1991), Jaksa (1995), Lacasse and Lamballerie (1995), Lacasse and Nadim (1996a), and Fenton (1999a,b). In 1999, Phoon and Kulhawy (1999a,b) decomposed uncertainties into spatial variability, measurement error, statistical uncertainty, and transformation uncertainty, which was of great influence on how we currently model uncertainties. Jaksa et al. (1999) investigated experimentally vertical and horizontal fluctuation scales by analyzing cone penetration tests (CPT) carried out in a stiff, overconsolidated clay. The most up-to-date review of the scale of fluctuation in random fields was conducted by Cami et al. (2020). Probabilistic descriptions of soil parameters derived from field and laboratory data and their application in stability analysis were also investigated by Christian et al. (1994), where the first-order second-moment (FOSM) approach was explored and applied to the design of embankment dams. Uzielli et al. (2007) provided a state-of-the-art review of approaches and methodologies for the quantification of soil variability, as well as selected examples of its utilization in reliability-based geotechnical design.

A breakthrough in practical adoption was made in 1997 by Low and Tang (1997), who provided a spreadsheet algorithm to implement the first-order reliability analysis method, which makes reliability analysis much less painful to perform than before. Their spreadsheet algorithm has since found wide applications in geotechnical engineering. Some of the research was published in the ASCE Proceedings "Uncertainty in the Geologic Environment: from Theory to Practice" (Shackelford et al. 1996). The body of work is presented elsewhere (Low 2021).

The last decade in the 20th century brought wider use of probabilistic approaches in geotechnical standards. Some examples of such applications follow here. The first example concerns the implementation of the reliability approach to geotechnical standards in Australia, e.g., Lo et al. (1992) and Li et al. (1993). Becker (1996) proposed incorporating reliability analysis into ultimate limit states of bearing capacity and sliding of shallow and deep foundations in an important study for the National Building Code of Canada. Such an approach aimed to provide a consistent design approach between geotechnical and structural engineers. Another example is a project of Eurocode 7 where reliability methods were not directly used but many elements of these standards were based on them, e.g., Orr and Farell (1999) and Orr (2000). The reliability-based design (RBD), whose goal is to calibrate the resistance (ultimate limit state) or deformation (serviceability limit state) factors in simplified design formats for a selected target reliability index, was also used in the AASHTO LRFD bridge design specifications (AASHTO, 1994). RBD was also applied to foundations for transmission line structures by Phoon et al. (1995, 2003a, 2003b). The application of reliability to calibrate a design guide to achieve an explicit target reliability index was likely first adopted for bridge foundations (Barker et al. 1991). The LRFD calibration approach has since been widely adopted in many AASHTO design problems (e.g., McVay et al. 1998, 2000; Rahman et al. 2002; Paikowsky et al. 2004, 2010; Allen 2005; Allen et al. 2005; Nowak et al. 2007; Zhang and Chu 2009a,b; Abu-Farsakh et al. 2009, 2013; Yang et al. 2010; Abu-Hejleh et al. 2011; Salgado et al. 2011; Smith et al. 2011; AbdelSalam et al. 2012; Ng and Fazia 2012; Penfield et al. 2014; Seo et al. 2015; Motamed et al. 2016; Bathurst et al. 2017; Yu et al. 2017; Haque and Abu-

Farsakh 2018; Tang and Phoon 2018; Kalmogo et al. 2019; Ng et al. 2019; Petek et al. 2020). Examples of rigorous reliability theory-based LRFD calibration of internal stability limit states for mechanically stabilized earth (MSE) walls can be found in the papers by Bathurst et al. (2019, 2021) and for soil nails by Lin and Bathurst (2019). Najjar and Gilbert (2009) further examined the effect of a lower-bound capacity in the LRFD design of deep foundations. Phoon et al. (1995) were the first to suggest using multiple resistance factors to accommodate different site conditions. The need to cater for diverse local site conditions and diverse local practices that grew and adapted over the years to suit these conditions was recognized by ISO2394:2015 (International Organisation for Standardization 2015) and the Canadian Highway Bridge Design Code (Canadian Standards Association 2014). A more detailed review of the historical evolution in geotechnical design philosophy and reliability-based design is provided by Phoon et al. (1995). This decade brought the general acceptance of RBD in geotechnical engineering among the geotechnical community. The Committee on Reliability Methods for Risk Mitigation in Geotechnical Engineering was convened by the National Research Council's Geotechnical Board in 1992 to examine the reasons for the seemingly slow acceptance of formal probabilistic methods in geotechnical engineering practice and to explore the potential for their wider use. The Committee's efforts to bridge theory and practice are notable (National Research Council 1995).

In the 1990s, along with the wider application of numerical models in geotechnical engineering, Bayesian methods were developed to calibrate the parameters of numerical models (e.g., Reddi and Wu 1991; Ledesma et al. 1996a,b). Honjo and his co-workers developed the extended Bayesian methods for calibrating geotechnical models using observed data from the field (e.g., Honjo et al. 1994; Honjo and Kashiwagi 1999). Gilbert (1999) suggested a first-order second-moment Bayesian method (FOSM) to help calibrate geotechnical models and geotechnical decision making, and the suggested method was later applied for calibrating a numerical flow and transport model based on model test data (Welker and Gilbert 2003). Angulo and Tang (1999) designed an optimal ground-water detection monitoring system using the Bayesian preposterior analysis. Gilbert and Tang (1995) highlighted the challenges in calibrating geotechnical model uncertainty when the model parameters are uncertain and discussed how such uncertainties can be considered using Bayesian methods.

In the 1990s, researchers at NGI contributed to reliability and risk analysis for offshore structures. Lacasse and Goulois (1989) investigated the uncertainty in the API (American Petroleum Institute) parameters for predictions of the axial capacity of driven piles in sand and collated the opinion of 40 international experts in assessing such uncertainties, providing a best-practice example of elicitation of expert judgment. Lacasse and Nadim (1996b) further investigated model uncertainty in pile axial capacity calculations from back-calculations of model tests and comparison of several methods of analyses. Nadim and Lacasse (1992) provided comparative examples of geotechnical stability analyses for offshore structures performed with the effective stress and total stress approaches on a contractant and a dilatant soil. They highlighted the importance of the probabilistic approach, showing that computed failure probability differed significantly for each approach, although the computed factors of safety for the dilatant material were nearly the same. Nadim and Gudmestad (1994) investigated the seismic reliability of a group of offshore platforms to quantify the probability that oil production must be stopped completely given the occurrence of a specific seismic event. Lacasse and Nadim (1994) highlighted the important role of reliability-based approaches and methods in the strengthening of the dialogue between different specialty areas in offshore engineering. Lacasse and Nadim (2007) provided an overview paper on probabilistic geotechnical analyses for offshore facilities, which illustrated a wide range of quantitative methods including event tree analysis, fault tree analysis, Bayesian updating, first-order second-moment (FOSM) method, Monte-Carlo simulations, Bayesian networks, first- and second-order reliability method (FORM and SORM), and system reliability analysis.

Nadim and Kvalstad (2007) contributed a keynote paper on risk assessment and management for offshore geohazards. This paper provided a comprehensive set of guidelines and best-practice approaches to identifying, quantifying, and managing geotechnical risks to offshore structures. Cassidy et al. (2015) presented a state-of-the-art review of deterministic and probabilistic advances in the analysis of spudcan foundation behaviour, highlighting the role of Bayesian reasoning in conjunction with the observational method. Researchers at NGI also contributed to the risk assessment of dams. Høeg (1996) contributed a state-of-the-art on the status of risk assessment for dams and proposed a simplified probabilistic risk analysis which was applied in the reevaluation and re-certification of rockfill dams, and to set priority on remedial measures. Vick (1997) summarized risk analysis practice in 11 countries, based on a dedicated survey (e.g., Vick and Stewart 1996).

2000-2010

At the beginning of the 2000s, two Terzaghi Lectures were delivered by researchers from the field of uncertainty quantification in geotechnical engineering. The first was given by Suzanne Lacasse in 2001, titled "Protecting society from landslides - the role of the geotechnical engineer". The second was given by John T. Christian in 2003, titled "Geotechnical Engineering Reliability: How well do we know what we are doing?". Baecher and Christian (2003) published an important book on Reliability and Statistics in Geotechnical Engineering, which covers the subject of risk and reliability in both practical and research terms. Vick (2002) published a book on subjective probability and expert elicitation, which is a relatively rare attempt to formalize engineering judgment within reliability and risk analysis.

In the 2000s, the RFEM method became a tool for many geotechnical problems. The use of RFEM for the bearing capacity problems was initiated by Griffiths and Fenton (2001) in their study on the bearing capacity of spatially random undrained clay. The study was later extended to cohesive-frictional soils by Fenton and Griffiths (2003) and the evaluation of slope stability by Fenton and Griffiths (2004) and Griffiths et al. (2009a). RFEM was later used for three-dimensional probabilistic settlement of a foundation by Fenton and Griffiths (2005), where the authors modeled soil as a three-dimensional medium with spatially random Young's modulus (E) and estimated the reliability of shallow foundations against serviceability-limit-state failure. In 2005, a stochastic approach to the problem of bearing capacity by the method of characteristics was proposed by Przewłócki (2005). The author modified the method of characteristics to consider the randomness of the soil medium in the bearing capacity problem. Jaksa et al. (2005) simulated "virtual sites" using 3D random fields to investigate the effectiveness of site investigation. This virtual site simulation was later extended to grounds with multiple soil layers and lenses by Crisp et al. (2021). The spatial variability impact on three-dimensional long slope failures using RFEM was investigated by Hicks et al. (2008) and Hicks and Spencer (2010). They showed that three failure modes are possible, depending on the ratio of the horizontal scale of fluctuation to the slope size. Griffiths et al. (2009b) showed that ignoring the spatial variability in the third direction as assumed in two dimensional analyses may underestimate the probability of failure of long slopes. One practical outcome of RFEM is the improved definition of a characteristic value. Eurocode 7 (Comité Européen de Normalisation 2004) defines the characteristic value of a geotechnical parameter as a "cautious estimate of the value affecting the occurrence of the limit state." "The value affecting the occurrence of the limit state" was originally thought to be the same as the classical sample mean for independent data or a spatial average for correlated data defined by Vanmarcke (1977a, 1977b). Hicks and Samy (2002) showed that RFEM is needed to simulate this effective property (called for "reliability-based characteristic value") for a spatially heterogeneous soil mass. The reason is that the limit state is controlled by a critical failure path that is not the same as an arbitrarily prescribed path. After all, the critical path is identified as the one with the smallest factor of safety. Tabarroki et al. (2020) provided a cheaper but approximate solution to the reliability-based characteristic value. The authors called it the mobilization-based characteristic value. Griffiths and Fenton (2007) and Fenton and Griffiths (2008) published two books on *Probabilistic Methods in Geotechnical Engineering* and *Risk Assessment in Geotechnical Engineering*, which present a thorough examination of the theories and methodologies available for probabilistic modelling and risk assessment in geotechnical engineering, spanning the full range from established single-variable and first-order reliability methods to random field finite element methods.

The 2000s decade brought further applications of reliability approaches to national standards and engineering practice. A good example is Japanese Geocode 21 (Honjo 2005; Honjo et al. 2010; JGS 2006). The reliability approach in geotechnical engineering became more accessible to practitioners (e.g., Duncan 2000) by some algorithm implementations in widely accessible software, e.g., spreadsheet algorithms developed by Low et al. (1998), Low and Tang (2007), Low et al. (2007), and Wang et al. (2010a).

The 2000s experienced a surge in the application of Bayesian methods in geotechnical engineering. For soil liquefaction potential assessment problems, the Bayesian methods have been widely used to develop new generation liquefaction potential assessment models considering both the aleatory and epistemic uncertainties (e.g., Cetin et al. 2002; Moss et al. 2006). Juang et al. (2002) suggested a Bayesian mapping method to estimate the liquefaction probability based on case histories more realistically. Zhang (2004) illustrated how to use pile proof load tests to verify the reliability of the design of pile foundations. Goh et. al. (2005) used the Bayesian neural network algorithm to model the relationship between the soil undrained shear strength, the effective overburden stress, and the undrained side resistance alpha factor for drilled shafts. Wu (2011) and Wu et al. (2007) illustrated how the Bayesian method can be used as a formal tool to implement the observational method in geotechnical engineering through an embankment construction problem. Other applications are presented in a book edited by Phoon (2008). Zhang et al. (2009) developed a Bayesian framework in which the geotechnical model uncertainty can be calibrated considering the presence of parameter and model uncertainties. The framework also provides the basis for probabilistic back-analysis (Zhang et al. 2010a, b), from which the distributions of several sets of material parameters can be updated. In a deterministic framework, only a limited number of parameters can be back-calculated depending on the number of constraints available. In the study by Yan et al. (2009), the Bayesian probabilistic approach for model class selection was used to revisit the empirical multivariate linear regression formula of the compression index. In 2010, Wang et al. (2010) developed the Bayesian framework in conjunction with cone penetration tests (CPT) to estimate the sand effective friction angle and with random field theory to model the inherent spatial variability. Traditionally, the application of Bayesian methods in geotechnical engineering heavily relied on the conjugate priors where the analytical solution to the posterior distribution is available. Along with the increase of processing power of personal computers, many efficient algorithms have been developed for solving complex Bayesian problems in statistics (e.g., Gelman et al. 2004; Givens and Hoeting 2005). Zhang (2009) reviewed different algorithms for Bayesian computation, clarified the relationships among different algorithms, and assessed the potential of different algorithms for application in geotechnical engineering. With Markov chain Monte Carlo (MCMC) methods, the posterior distribution can be obtained even without the conjugate prior assumption. Ching and Chen (2007) developed the transitional Markov chain Monte Carlo (TMCMC) method that can solve complex Bayesian problems without specifying the proposal distribution. MCMC methods later become one of the main algorithms for solving complex Bayesian problems in the 2010s.

The 2000s is also a decade with wider application of artificial neural networks to geotechnical applications, e.g., Shahin et al. (2001), Shahin et al. (2008), and Jaksa et al. (2008). In the 2000s, the use of more and more sophisticated numerical methods in conjunction with the estimation of uncertainty in geotechnics increased the demand for data from geotechnical surveys and led to the fact that there were more and more of these data, but the possibility of using them was very limited. This problem became even more acute in the next decade. Mitchell and Kopmann (2013) spoke of the availability of big data sets and the challenge to make sense of this wealth of information: “In fact the easy access to such vast amounts of information about most of the topics included in this report, the evaluation of its validity and importance, and deciding which of it should be included was one of the major challenges faced by the authors, and it provided an excellent example of the ‘information overload’ problem.”

It is worthwhile to point out that the Geotechnical Safety Network (GEOSNet) was established in 2006 with the intent to: (1) expand the base of participation in industry and government agencies, (2) raise awareness in practice/education and expedite the transfer of knowledge between research and practice/education, (3) promote sharing of information on the development of geotechnical design codes between countries, (4) promote liaison with related committees within and without the geotechnical engineering community, (5) promote research and practice on assurance of geotechnical safety to keep pace with advancements in numerical methods, risk management methods, materials/equipment, and construction methods, (6) stimulate more discussions on safety issues in complex projects at design, construction, maintenance, and other stages, and (7) encourage engineers to embrace uncertainties and risks more explicitly and more systematically in practice and education. Seven international symposiums (International Symposium on Geotechnical Safety and Reliability; ISGSR) have been organized thus far in Shanghai, Gifu, Munich, Hong Kong, Rotterdam, Denver, and Taipei (https://geosnet.geoengineer.org/).

At around the same time, the first international journal dedicated to geotechnical risk and reliability entitled “Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards” was established in 2007. As pointed out in its first Editorial (Phoon et al. 2007), this journal was motivated by the recognition that “uncertainties associated with geomaterials (soils, rocks, snow), geologic processes and anthropogenic actions can be large and complex. These uncertainties play an important role in the assessment of hazard and risk and in the management of risk” and research needs to be promoted and communicated because “significant theoretical and practical challenges remain for quantifying the uncertainties and developing sustainable risk management methodologies that are attractive to decision-makers and stakeholders.” Georisk has since published 15 volumes, 13 special issues, and 5 spotlight articles. It was accepted by SCI in 2020 and received an impact factor of 3.868 in 2021.

2010-present

In the 2010s, researchers at NGI contributed several review/keynote papers regarding geotechnical risk and reliability. In her Rankine Lecture, Lacasse (2015) investigated the conceptual and operational connection between geotechnical practice and risk management and highlighted the importance of implementing concepts of hazard, risk, and reliability in routine analyses. Lacasse (2016) illustrated the use of the reliability and risk concepts with "real life" case studies, specifically for situations encountered in Nordic environments. The paper provided calculation examples from a wide realm of geotechnical problems, including avalanche, railroad safety, mine slopes, and soil investigations. In the

Third Suzanne Lacasse Honour Lecture, Nadim (2017) contributed a state-of-the-art paper focusing on concepts such as hazard, exposure, vulnerability, risk, risk management, acceptable risk, and reliability-based geotechnical design. Researchers at NGI also contributed to the risk and reliability of dams. Lacasse et al. (2019) provided an updated overview of basic concepts of reliability-based approaches applied to dams and illustrated their use with three case studies. This recent state-of-the-art paper discussed the strengths of reliability-based analyses and key issues such as tolerable and acceptable risk, the meaning of factor of safety, the targets for margins of safety, and the selection of characteristic values for analysis.

Mechanics-based methods for quantitative risk assessment of slope failure attracted a lot of attention in this decade. Huang et al. (2013) proposed a quantitative risk assessment approach of landslide in spatially variable soils wherein the consequences were assessed individually for each potential failure mode. Zhang et al. (2016) published a monograph titled *Rainfall-Induced Soil Slope Failure: Stability Analysis and Probabilistic Assessment*, which systematically introduced the deterministic and probabilistic methods for analysis of rainfall-induced slope failures as well as their applications in quantitative risk assessment. Wang et al. (2016a) used the random material point method (RMPM) for quantitative risk assessment of slopes where post-failure behaviour can be considered. Their results showed that RMPM provides a much wider range of solutions, in general increasing the volume of material in the failure compared to RFEM solutions. Methods have been developed for assessing the annual failure probability of slopes subjected to seismic shaking (e.g., Wang and Rathje 2018; Huang et al. 2018; Macedo et al. 2018; Zhang et al. 2021), which is essential for risk assessment. Zhang et al. (2021) developed a mechanics-based method to estimate the annual probability of slope failure caused by rainfall infiltration. Moreover, some efficient approaches for three-dimensional bearing capacity estimation for spatially variable soils were proposed, e.g., Chwała (2019) and Li et al. (2021). In the meantime, works on estimating scales of fluctuation remained active, e.g., Lloret-Cabot et al. (2014), Pieczyńska-Kozłowska et al. (2017), Ching et al. (2018), and Cami et al. (2020). The decade brings also some effort to optimization of site investigation programs that maximize robustness and minimize site investigation effort or propose the optimal location of soil soundings (e.g., Li et al. 2016a; Gong et. al 2017; Chwała, 2020; Jiang et al. 2020; Crisp et al. 2020). Recently, some researchers turned towards the application of probabilistic methods in conjunction with more complex subsoil models than the Coulomb-Mohr constitutive soil model. These studies used the Hardening-Soil model (e.g., Sert et al. 2016; Luo et al. 2018; Kawa et al. 2021), or the modified Cam Clay model (e.g., Savvides and Papadrakakis 2021).

The 2010s is also a decade with wider application of risk assessment and management combining the reliability and the consequence evaluation in particular with the fields of underground infrastructure system (e.g., tunnels). The International Tunnelling Association ITA (Eskesen et al. 2004) has published a guideline for tunnelling risk management. A notable example of national codes was published in China which is edited by Zhang and Huang (2010). The quantitative risk analysis (QRA) of a tunnel and cut-slope as a system was proposed by Li et al. (2010) considering the failure probability of a risk event occurring in a space domain at a specific time using quantitative vulnerability analysis. Wireless sensor network (WSN)-based risk sensing and monitoring was developed and applied to the real-time risk control in tunnel engineering (Huang et al. 2017). It is followed by a national code of the WSN based risk sensing for infrastructure published in China (Huang 2021). The real time risk sensing along a 20.4km-long metro tunnel lining was applied to the Shanghai metro since 2015. The project is still on-going with at least 6 years continuous monitoring of the operational risk of the metro tunnel (Huang et al. 2017). The next step for risk management of this underground tunnel is to enhance the resilience of the infrastructure (Huang and Zhang, 2016; Zhang et al. 2018).

Due to the continuing increase in computational power, random field geotechnical numerical analyses became increasingly feasible in the 2010s. With the consideration of spatial variability, the failure path and failure mechanisms became more complex and can be correctly treated as unknown prior to the analysis. To address different potential failure modes, system reliability problems are drawing attention in geotechnical reliability and risk, particularly for slope reliability analysis (e.g., Ching et al. 2009; Huang et al. 2010; Griffiths et al. 2011; Wang et al. 2011; Zhang et al. 2011; Li et al. 2011). Significant challenge encountered in slope system reliability analysis is the prohibitive computational cost when a large number of potential slip surfaces are considered. To address the computational issue, it has been found that the system reliability of a slope is often controlled by a small number of representative slip surfaces (Zhang et al. 2011), and a sub-system comprised of representative slip surfaces can be identified and used to approximate the original and complete system (Zhang et al. 2011; Li et al., 2013). The accuracy of slope system reliability based on representative slip surfaces depends on the correlation of performance functions of different failure modes, which is problem dependent and unknown. This limitation can be alleviated using multiple response surface methods (MRSM) (e.g., Li et al. 2015), where each component performance function is represent by a user-defined response surface. After the response surfaces of all components are constructed, it is trivial to perform system reliability analysis with negligible extra computational cost. Li et al. (2016b) reviewed response surface methods for slope problems. Alternatively, advanced Monte Carlo simulation methods were also developed to tackle computational difficulties in geotechnical system reliability analysis and risk assessment, such as importance sampling (e.g., Ching et al. 2009), subset simulation (e.g., Wang et al. 2011; Li et al. 2016c; Huang et al. 2017), and adaptive Monte Carlo simulation (Liu et al. 2020). These advanced Monte Carlo simulation methods were shown to provide efficient and robust solutions to geotechnical system reliability problems. Luo and Bathurst (2018a,b) extended the RFEM to include reinforced slopes and embankments with spatial variability of soil strength. Javankhoshdel et al. (2017) coined the term random limit equilibrium method (RLEM) as an alternative approach to RFEM for probabilistic slope stability analysis of slopes with spatial variability.

The importance of spatial variability, measurement error, statistical uncertainty, and transformation uncertainty for several geotechnical problems in reliability-based design (RBD) was discussed by Honjo (2011). The 2010s decade brought further migration of RBD knowledge and experiences to national standards, e.g., the Canadian Highway Bridge Design Code (Fenton et al. 2016; CAN/CSA-S6-14 2014). In 2015, the 4th edition of the ISO international standard was published, i.e., "General Principles on Reliability for Structures" (ISO 2394:2015), where Annex D was dedicated to the reliability of geotechnical structures (Phoon and Retief 2016). Reviews of semi-probabilistic reliability-based design and direct probability-based design methods are provided by Phoon and Ching (2016) and Wang et al. (2016b), respectively. Uzielli et al. (2012) proposed a simple probabilistic, design code-format correlation for assigning design values of effective friction angle of clean sands from in-situ tests incorporating all sources of geotechnical uncertainty, including transformation uncertainty. Cao et al. (2019a) provided a review of Monte Carlo simulation-based methods for full probabilistic design and emphasized values of Monte Carlo samples for geotechnical reliability-based design. CPT-based reliability-based methods were also contributed. Uzielli and Mayne (2011) developed a CPT-based method for the design of vertically loaded shallow footings on sand at the serviceability limit state. In 2014, Juang et al. (2013a) and Juang and Wang (2013) proposed the idea of robust geotechnical design, where the robust design of a braced excavation system (including soil, wall, and support) was formulated as a multi-objective optimization problem, in which the variation of the maximum wall deflection (a signal of the design robustness) and the cost were optimized with the strict safety constraints. In the 2010s further development of opensource software that can be used for uncertainty quantification continued. A good example is OpenCossan software (Patelli et al. 2014). The software was successfully used in a variety of geotechnical applications (e.g., He et al. 2020).

In the study by Christian and Baecher (2011), ten unresolved problems in geotechnical risk and reliability were identified. Among them was the connection between the observational method and Bayesian updating. In the 2010s, probabilistic back-analysis of slope failure based on Bayesian methods found wide application (i.e., Zhang et al. 2010a, b; Ering and Sivakumar Babu 2016; Jahanfar et al. 2017). The Bayesian method solved with MCMC simulation was also used to implement the observational method for the design and construction of deep excavation problems (e.g., Juang et al. 2013b), embankments built on soft soils (Zheng et al. 2018), and for the prediction of peak resistance of a spudcan penetrating stiff-over-soft clay (Uzielli et al. 2017) and sand-over-clay (Li et al. 2018). Traditionally, the Bayesian method is often used in geotechnical engineering to back-analyze random variables. Yang et al. (2018) developed Bayesian methods such that random fields can also be back-analyzed using monitored data. The Bayesian method has been recognized as the main tool for interpreting site investigation data (e.g., Wang et al. 2016c; Juang et al. 2019). It allows for systematic accumulation and updating of site knowledge with increasing data and quantifies site uncertainty to reflect the state of knowledge on site (e.g., Cao et al. 2016). Huang et al. (2018) used Bayesian updating to integrate cone penetration test (CPT) with multi-channel analysis of surface wave (MASW) data for geotechnical site characterisation. It has been noted since the work by Baecher and Rackwitz (1982) that geotechnical data can contain within-site and cross-site variabilities. Extending such an idea, geotechnical data of different groups may also have inter-group and intra-group variability. Recent studies have focused on how to model such variabilities using hierarchical Bayesian models which improve knowledge for one group from data from other groups (e.g., Zhang et al. 2014, 2016; Bozorgzadeh and Bathurst 2020; Ching et al. 2021a; Xiao et al. 2021). The Bayesian approach was recently used by Chen et al. (2020) to update knowledge about the design model uncertainties for fixed steel offshore platforms by using data obtained for large soil-structure systems during and after major hurricanes in the Gulf of Mexico. Bayesian methods were also adopted to the purpose of geotechnical design.

Possible data-centric future

Bayesian thinking (e.g., Baecher 2017) has been suggested as a foundation for data-driven approaches developed in the 2010s. In 2017, to promote the use of Bayesian methods in geotechnical engineering, the joint working group of TC205 and TC304 published a report to summarize the techniques, advantages, and the application examples of Bayesian methods in geotechnical engineering. Juang and Zhang (2017) provided a practical guide with detailed illustrative examples to learn Bayesian methods in the context of geotechnical engineering. In 2021, Gregory Baecher delivered the Terzaghi Lecture titled “Geotechnical systems, uncertainty, and risk”, where he addressed the importance of the Bayesian approach in geotechnical engineering. Gregory Baecher, Herbert Einstein, Wilson Tang, and TH Wu were among the early Bayesianists in geotechnical engineering. Bayesian methods are now important for machine learning (Phoon and Zhang 2022).

The end of the 2010s decade brought new interest in data itself. Attempts were made to collect characterization of geotechnical data from worldwide surveys and research. Phoon et al. (2019) coined the term Big Indirect Data (BID) to refer to any data that are potentially useful but not directly applicable to the decision at hand. Databases containing property or load test data from multiple sites would belong to BID. Recent reviews on uncertainty representation of geotechnical design parameters and statistical characterization of multivariate geotechnical data were provided by Phoon et al. (2016) and Ching et al. (2016), respectively. Such a database (Project 304 dB, TC304, 2019) is open to the public and is still being updated. Although the main attention was focused on soil properties, significant progress was made in compiling performance data, particularly foundation load test databases (Phoon and Tang 2019; Tang and Phoon 2021). Tang and Phoon (2021) compiled the largest load test database to date, covering various foundation types (shallow foundation, offshore spudcan in layered soils, driven and drilled shaft, and helical pile) in a wide range of ground conditions (clay, silt, sand, gravel and rock) and presented a comprehensive survey of the performance databases for other geo-structures (e.g., soil nail/mechanically stabilized earth walls, slope, plate/anchor, braced excavation).

With the increasing availability of data and the advent of digital transformation, the 2010s decade also brought further usage and development of machine learning approaches in geotechnical reliability and risk analysis, e.g., for slope reliability analysis by Kang et al. (2016), approach for rational and objective interpretation of the soil property profile with quantification of the associated statistical uncertainty (Wang and Zhao, 2017), underground stratification and soil classification (e.g., Cao and Wang, 2013; Depina et al. 2016; Cao et al. 2019b; Wang et al. 2019; Xiao et al. 2021), and landslide susceptibility assessment (Wang et al. 2021a,b). A short review of deep learning is presented by Zhang et al. (2021). Jong et al. (2021) reviewed application of AI techniques to underground soil-structure interaction problems such as characterization of soils and rocks, pile foundations, deep excavations and tunnelling. One major challenge (termed "site recognition challenge") is to quantify "site uniqueness", directly or indirectly, so that big indirect data (BIDs) can be combined with sparse site-specific (local) data in a manner sensitive to site differences (Phoon et al. 2021; Ching et al. 2021a). It is likely that geotechnical data and its role in decision making will be broadened and deepened in the near future in light of rapid advancements in machine learning and artificial intelligence in the past 5 or so years.

Phoon and Ching (2021) and Phoon et al. (2022a) referred to the growing body of research on databases and data-driven methods as "data-centric geotechnics". The central tenet in data-centric geotechnics is that data has value as long as it is not fake. The challenge is to draw useful inferences for decision-making from real world data over the entire lifecycle covering design, construction, operation, maintenance, and decommissioning. Phoon et al. (2021) termed real world data as "ugly data" to contrast with high quality data demanded by the existing deterministic design paradigm. In the 4th Suzanne Lacasse Lecture, Phoon et al. (2019) suggested that the ugly attributes of real world site investigation data can be summarized using the mnemonic MUSIC-X (Multivariate, Uncertain, Unique, Sparse, Incomplete, potentially Corrupted, and spatially variable X). Phoon et al. (2021) pointed that there are other ugly or uglier attributes in rock engineering, because of mixed data types (nominal/ordinal/interval/ratio). The genetic group – sedimentary, igneous, and metamorphic – is an example of nominal data. The joint set number (J_n) in the Q-system is an example of ordinal data. More research is needed to understand data and develop appropriate data-driven methods for rock engineering. The ability to draw useful engineering insights from ugly data ("ugly data challenge") is considered a fundamental problem in data-centric geotechnics. One problem that has attracted attention is site characterization because it is a cornerstone of geotechnical engineering. Ching and Phoon (2019) applied a variant of the powerful Gaussian Process Regression (GPR) method to construct a multivariate probability density function for MUSIC data. This is the first time a probability density function can be constructed in a multivariate setting where the data is both incomplete and sparse, although this setting is commonly encountered in site investigation for a routine project. The Bayesian approach is necessary. This GPR-MUSIC method has since been extended to cover 1D spatial variability (GPR-MUSIC-X, Ching and Phoon 2020) and 3D spatial variability (GPR-MUSIC-3X, Ching et al. 2021b). Wang et al. (2021c) presented a non-parametric approach based on Bayesian compressive sampling.

Compressive sampling is distinct from the classical Fourier transform. It can represent a signal with frequencies beyond what is permissible by the sampling interval (Nyquist–Shannon sampling theorem). This is highly advantageous for sparse site-specific data. Shuku et a. (2020) focused on the detection of stratigraphic boundaries using a sparse Bayesian lasso method. The lasso method is by far the best in detecting sudden changes in the sounding profile, which is exactly what is needed for mapping stratigraphy. The key practical purpose of these data-driven methods is to estimate the properties and/or stratigraphic boundaries at unobserved locations based on the typical MUSIC-3X data measured at highly limited locations at a single site. Hu et al. (2021) suggested a method to assess the effectiveness of geotechnical site investigation programs for design of slopes. Phoon et al. (2021) termed this exercise as "data-driven site characterization" (DDSC) and defined DDSC as "any site characterization methodology that relies solely on measured data, both site-specific data collected for the current project and existing data of any type collected from past stages of the same project or past projects at the same site, neighbouring sites, or beyond." Another challenge embedded in DDSC that is distinct from the "ugly data challenge" is the "site recognition challenge". The purpose of this challenge is to improve the estimation at unobserved locations using both site-specific data and data from "similar" sites. The latter data is not sparse as it is assembled from multiple sites (termed "Big Indirect Data" or BID in Phoon et al. 2019). An engineer regularly combines site-specific data with data from other relevant sources through an appreciation of regional geology and experiences gathered from past projects, but there is no data-driven method that can do this comprehensively and satisfactorily to date. The Hierarchical Bayesian model was found to be promising (Ching et al. 2021a). Shi and Wang (2021) proposed a novel iterative convolution eXtreme Gradient Boosting model (IC-XGBoost) that can interpolate a subsurface geological cross-section from limited site-specific borehole data and a training geological cross-section obtained from previous projects with similar geological profiles. Phoon and Ching (2021) summarized progress to date on DDSC and postulated that "DDSC may one day evolve into an artificial intelligence (AI) that can emulate human learning and experience building". Phoon (2020) called this longer term project AlphaGeo. Phoon et al. (2021) imagined an artificial intelligence that can mimic human learning to be equivalent to a 'super engineer' that has gained and continues to gain from the pooled experiences of all human engineers around the world. Since it is already known that an experienced engineer makes better judgment than a novice with no experience, one can speculate that such a 'super engineer' with access to extensive databases, capable of detecting site differences through data-driven methods … and capable of moderating its predictions by drawing upon relevant past human experiences will be making better site-specific predictions than those from classical statistical methods moderated by 'reality checks' from a single engineer."

Recognising the potential and growing importance of modern data-driven methods, the journal *Georisk* has expanded its aim and scope to embrace the rapid advancements in machine learning, artificial intelligence, and other data-driven methods and their applications to enhance our design, construction, and decision-making abilities.

To meet the increasing educational need to train professionals with expertise on geotechnical reliability, Zhang et al. (2021) developed a textbook titled "*Geotechnical Reliability Analysis: Theories, Methods, and Algorithms*". A MOOC (Massive Open Online Courses) on *Probability Analysis in Civil Engineering* has been launched by TC304 in the ISSMGE Virtual University to teach reliability theory online.

Remarks

What seems very interesting, in each decade there was a struggle to increase the realism, generality, and numerical efficiency of proposed approaches, regardless of how fast computers we had. It looks like it will stay that way in the future. Nevertheless, with the progress we made, we are capable of solving more and more complex geotechnical reliability problems, making a greater impact in the profession. As Phoon (2020) exhorted in the 10th Lumb Lecture, "the value of geotechnical data is significantly under-appreciated and not fully exploited for decision making. Our data is 'dark' in the sense that it is stored primarily for compliance purposes, rather than shared and actively mined for insights that can inform future decision making. The world is being revolutionised by new and powerful ways of collecting, sharing, analysing, and monetizing data. Clearly, there is a pressing need for the geotechnical engineering community to engage in this digital transformation". The current state of play was discussed in the ISSMGE TC309/TC304/TC222 Third Machine Learning in Geotechnics Dialogue (3MLIGD) (Phoon et al. 2022b). A review of machine learning in geotechnics is presented by Phoon and Zhang (2022).

The ground is complex, because of its natural origin. Important gaps in our knowledge such as geologic history, ground water flow, and possibly recent man-made disturbances from construction or other sources are present in all geotechnical projects. Geotechnical engineering decision making remains a "calculated" risk informed by limited data, incomplete knowledge, and imperfect theories. There is no doubt that insights provided by mechanical, probabilistic, or other analyses need to be moderated by engineering judgment, but it is unclear how engineering judgment should evolve as the capabilities of computational tools and digital technologies continue to grow in power rapidly.

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References

AASHTO. 1994. *AASHTO LRFD Bridge Design Specifications*, SI units, First Edition. American Association of State Highway and Transportation Officials.

Abdelsalam, S.S., Sritharan, S., Suleiman, M.T., Ng, K.W., and Roling, M.J. 2012. *Development of LRFD Design Procedures for Bridge Pile Foundations in Iowa, Vol. 3: Recommended Resistant Factors with Consideration to Construction Control and Setup.* Report No. IHRB Projects TR-584. Ames, IA: Iowa Department of Transportation.

Abu-Farsakh, M.Y., Chen, Q.M., and Haque, M.N. 2013. *Calibration of Resistance Factors for Drilled Shafts for the New FHWA Design Method*. Report No. FHWA/LA.12/495. Baton Rouge, LA: Louisiana Transportation Research Centre.

Abu-Farsakh, M.Y., Yoon, S.M., and Tsai, C. 2009. *Calibration of Resistance Factors Needed in the LRFD Design of Driven Piles.* Report No. FHWA/LA.09/449. Baton Rouge, LA: Louisiana Transportation Research Center.

Abu-Hejleh, N., DiMaggio, J.A., Kramer, W.M., Anderson, S., and Nichols, S. 2010. *Implementation of LRFD Geotechnical Design for Bridge Foundations.* Report No. FHWA NHI-10-039. Washington, DC: National Highway Institute, Federal Highway Administration, and U.S. Department of Transportation.

Allen, T.M. 2005. *Development of Geotechnical Resistance Factors and Downdrag Load Factors for LRFD Foundation Strength Limit State Design.* Report No. FHWA-NHI-05-052. Washington, DC: National Highway Institute, Federal Highway Administration, and U.S. Department of Transportation.

Allen, T.M., Nowak, A.S., and Bathurst, R.J. 2005. *Calibration to Determine Load and Resistance Factors for Geotechnical and Structural Design*. Transportation Research Circular E-C079. Washington, DC: Transportation Research Board of the National Academies.

Alonso E.E. 1976. "*Risk analysis of slopes and its application to slopes in Canadian sensitive clays*." Geotechnique, 26(3), 453-472.

Ang, A.H.-S. and Tang, W.H. 1975. *Probability Concepts in Engineering Planning and Design*, Vol. 1, Basic Principles, John Wiley, New York.

Ang, A.H.-S. and Tang, W.H. 1984. *Probability Concepts in Engineering Planning and Design,* Vol. 2, Decision, Risk and Reliability, John Wiley & Sons, Inc., New York.

Ang, A.H.-S. and Tang, W.H. 2007. *Probability Concepts in Engineering*. John Wiley, New York.

Angulo, M. and Tang, W.H. 1999. "*Optimal ground-water detection monitoring system design under uncertainty.*" Journal of Geotechnical and Geoenvironmental Engineering, 125(6), 510-517.

Baecher, G.B. 1985. *Geotechnical Error Analysis, Special Summer Course 1.60s*. MIT, Cambridge, USA.

Baecher, G.B. 1987. *Geotechnical Risk Analysis User's Guide*. FHWA/RD-87-011, Federal Highway Administration, McLean, USA.

Baecher, G.B. 2016. "*Uncertainty in dam safety risk analysis*." Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 10(2), 92-108.

Baecher, G.B. 2017. *Bayesian thinking in geotechnics*. Proceeding of GeoRisk 2017, Denver.

Baecher, G.B. and Christian, J.T. 2003. *Reliability and Statistics in Geotechnical Engineering*. Wiley, Chichester.

Baecher, G.B. and Christian, J.T. 2019. *"TH Wu and the origins of geotechnical reliability*." Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 13(4), 242-246.

Baecher, G.B. and Ingra, T.S. 1981. "*Stochastic FEM in settlement predictions*." Journal of Geotechnical Engineering, 107(4), 449-463.

Baecher, G.B. and Ladd, C.C. 1985. *Reliability Analysis of Stability of Embankments of Soft Clays*. Special Summer Course 1.60s, MIT, Cambridge, USA.

Baecher, G.B., Pate, M.E., and De Neufville, R. 1980. "*Risk of dam failure in benefit-cost analysis.*" Water Resources Research, 16(3), 449-456.

Baecher, G.R. and Rackwitz, R. 1982. "*Factors of safety and pile load tests*." Int. J. Numer. Anal. Meth. Geomech., 6(4), 409-424.

Barker, R.M., Duncan, J.M., Rojiani, K.B., Ooi, P.S.K., Tan, C.K., and Kim, S.G. 1991. *Manuals for Design of Bridge Foundations*, NCHRP Report 343, Transportation Research Board, Washington, 1991.

Bathurst, R.J., Allen, T.M., Lin, P., and Bozorgzadeh, N. 2019. "*LRFD calibration of internal limit states for geogrid MSE walls.*" Journal of Geotechnical and Geoenvironmental Engineering, 145(11), 04019087

Bathurst, R.J., Bozorgzadeh, N., and Allen, T.M. 2021. "*LRFD calibration of internal limit states for MSE walls using steel strip reinforcement.*" Journal of Geotechnical and Geoenvironmental Engineering, 147(12), 04021156.

Bathurst, R.J., Javankhoshdel, S., and Allen, T.M. 2017. "*LRFD calibration of simple soil-structure limit states considering method bias and design parameter variability*." Journal of Geotechnical and Geoenvironmental Engineering, 143(9), 04017053.

Becker, D. 1996. "*Eighteen Canadian geotechnical colloquium: limit states design for foundations, Part II. Development for the national building code of Canada*." Canadian Geotechnical Journal, 33(6), 984-1007.

Benjamin, J.R. and Cornell, C.A. 1970. *Probability, Statistics, and Decision for Civil Engineers*. New York: McGraw-Hill.

Biernatowski, K. 1966a. *"Satateczność fundamentów cz. I – Zastosowanie rachunku prawdopodobieństwa dla oceny współczynników bezpieczeństwa przy sprawdzaniu fundamentów*." Archiwum Hydrotechniki, 13(2), 239-262.

Biernatowski, K. 1966b. *"Stateczność fundamentów cz. II – Stateczność na obrót i przesunięcie w ujęciu probabilistycznym.*" Archiwum Hydrotechniki, 13(2), 283-302.

Biernatowski, K. 1968. "*Stability of slopes in probabilistic solutions."* Proc. of the 7th Int. Conf. on Soil Mechanics and Foundation Engineering, Mexico City, 2, 527-530.

Biernatowski, K. 1972. "*Stability of rigid structures in probabilistic formulation.*" Proc. of the 5th European Conf. on Soil Mechanics and Foundation Engineering, 1, 111-116.

Biernatowski, K. and Puła, W. 1988. "*Probabilistic analysis of the stability of massive bridge abutments using simulation methods*". Structural Safety, 5, 1-15.

Bowles, D.S. and Ko, H.J. 1984. *Probabilistic Characterization of Soil Properties: Bridge between Theory and Practice*, ASCE.

Bozorgzadeh, N. and Bathurst, R.J. 2020. "*Hierarchical Bayesian approaches to statistical modelling of geotechnical data*." Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards. https://doi.org/10.1080/17499518.2020.1864411

Brown, E. T. 2012. "Risk assessment and management in underground rock engineering—an overview". Journal of Rock Mechanics and Geotechnical Engineering, 4(3), 193-204.

Brząkała, W. 1988. "*Kandaurov's stress solution for granular media in elasticity theory context*." Bulletin of the Polish Academy of Sciences. Technical Sciences, 36(7/9), 397-406.

Brząkała, W. and Puła, W. 1996. "*A probabilistic analysis of foundation settlements.*" Computers and Geotechnics, 18(4), 291-309.

Cami, B., Javankhoshdel, S., Phoon, K.K., and Ching, J. 2020. "*Scale of fluctuation for spatially varying soils: estimation methods and values.*" ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 6(4), 03120002.

Canadian Standards Association. 2014. *Canadian Highway Bridge Design Code*. CAN/CSA-S6-14, Mississauga, Ont.

Cao, Z. J., and Wang, Y. 2013. "*Bayesian approach for probabilistic site characterization using cone penetration tests*." Journal of Geotechnical and Geoenvironmental Engineering, 139(2), 267-276.

Cao, Z.J., Gao, G.H., Li, D.Q., and Wang, Y. 2019a. "*Values of Monte Carlo samples for geotechnical reliability-based design.*" The Seventh International Symposium on Geotechnical Safety and Risk (ISGSR 2019), Taipei, Taiwan.

Cao, Z.J., Wang, Y., and Li, D.Q. 2016. "*Site-specific characterization of soil properties using multiple measurements from different test procedures at different locations* – A Bayesian sequential updating approach." Engineering Geology, 211, 150–161.

Cao, Z.J., Zheng, S., Li, D. Q., and Phoon, K.K. 2019b. "*Bayesian identification of soil stratigraphy based on soil behaviour type index*." Canadian Geotechnical Journal, 56(4), 570-586.

Casagrade, A. 1965. "*Role of the 'calculated risk' in earthwork and foundation engineering.*" Journal of the Soil Mechanics and Foundations Division, 91(4), 1-40.

Cassidy, M.J., Li, J., Hu, P., Uzielli, M., Lacasse, S. 2015. "Deterministic and probabilistic advances in the analysis of spudcan behaviour". The Third International Symposium on Frontiers in Offshore Geotechnics (ISFOG 2015), Oslo, Norway.

Cetin, K.O., Kiureghian, A.D., and Seed, R.B. 2002. "*Probabilistic models for the initiation of seismic soil liquefaction.*" Structural Safety, 24, 67-82.

Chen, J., Gilbert, R., Ku, A., Chen, J.Y., Marshall, P.W. 2020. "*Calibration of model uncertainties for fixed steel offshore platforms based on observed performance in Gulf of Mexico hurricanes*." Journal of Geotechnical and Geoenvironmental Engineering, 146(6), 04020039.

Ching, J. and Chen, Y.C., 2007. "*Transitional Markov chain Monte Carlo method for Bayesian model updating, model class selection, and model averaging*." Journal of Engineering Mechanics, 133(7), 816-832.

Ching, J. and Phoon, K. K. 2013. "Mobilized shear strength of spatially variable soils under simple stress states". Structural Safety, 41, 20-28.

Ching, J. and Phoon, K.K. 2019. "*Constructing site-specific multivariate probability distribution model using Bayesian machine learning.*" Journal of Engineering Mechanics, 145(1), 04018126.

Ching, J. and Phoon, K.K. 2020. "*Constructing a site-specific multivariate probability distribution using sparse, incomplete, and spatially variable (MUSIC-X) data*". Journal of Engineering Mechanics, 146(7), 04020061

Ching, J., Li, D.Q., and Phoon, K. K. 2016. "*Statistical characterization of multivariate geotechnical data*." Chapter 4, Reliability of Geotechnical Structures in ISO2394, CRC Press/Balkema, 89-126.

Ching, J., Phoon, K.K., and Hu, Y.G. 2009. "*Efficient evaluation of reliability for slopes with circular slip surfaces using importance sampling*". Journal of Geotechnical and Geoenvironmental Engineering, 135(6), 768-777.

Ching, J., Phoon, K.K., Yang, Z.Y., and Stuedlein, A.W. (2021b). *"Quasi-site-specific multivariate probability distribution model for sparse, incomplete, and three-dimensional spatially varying soil data."* Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, https://www.tandfonline.com/doi/full/10.1080/17499518.2021.1971256

Ching, J., Wu, S., and Phoon, K. K. 2021a. *"Constructing quasi-site-specific multivariate probability distribution using hierarchical Bayesian model*." Journal of Engineering Mechanics, *ASCE, 147 (10), 04021069*

Chowdhury, R., Tang, W.H., and Sidi, I. 1987. "*Reliability model of progressive slope failure.*" Geotechnique, 37(4), 467-481.

Christian, J.T. 2003. "*Geotechnical engineering reliability: How well do we know what we are doing?*" Thirty-Ninth Terzaghi Lecture presented at the ASCE 2003 Civil Engineering Conference and Exposition, Nashville, Tenn., November 2003.

Christian, J.T. and Baecher, G.B. 2011. "*Unresolved problems in geotechnical risk and reliability*." Geo-Risk 2011: Risk Assessment and Management.

Christian, J.T. and Baecher, G.B. 2015. "D. W. Taylor and the Foundations of Modern Soil Mechanics." Journal of Geotechnical and Geoenvironmental Engineering, 141(2), 02514001

Christian, J.T., Ladd, C.C., Baecher, G.B. 1994. "*Reliability applied to slope analysis*." Journal of Geotechnical Engineering, 120(12), 2180-2207.

Chwała, M. 2019. "*Undrained bearing capacity of spatially random soil for rectangular footings*." Soils and Foundations, 59(5), 1508-1521. https://doi.org/10.1016/j.sandf.2019.07.005

Chwała, M. 2020. M. "*Soil sounding location optimisation for spatially variable soil*". Géotech. Lett., 10(3), 409-418. https://doi.org/10.1680/jgele.20.00012

Comité Européen de Normalisation. 2004. *EN 1997-1: Geotechnical Design – Part 1: General Rules*.

Contreras, L.-F., and Brown, E. T. 2018. "Bayesian inference of geotechnical parameters for slope reliability analysis." XIV Congreso Internacional de Energia y Recursons Minerales, Seville, 29.

Cornell, C. A. 1971. "First-order uncertainty analysis of soil deformation and stability." Proc. 1st Int. Conf. Application of Statistics and Probability in Soil and Structural Engineering, Hong Kong, 130–143.

Corotis, R.B., Azzouz, A.S., and Krizek, R.J. 1975. "*Statistical evaluation of soil index properties and constrained modulus.* " Proc. of the 2nd Int. Conf. of Statistics and Probability in Soil and Structural Engineering (ICASP), Aachen, 273-293.

Crisp, M.P., Jaksa, M., and Kuo, Y. 2021. Characterising site investigation performance in multiple-layer soils and soil lenses. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 15(3), 196-208.

Crisp, M.P., Jaksa, M., Kuo, Y. 2020. "*Optimal Testing Locations in Geotechnical Site Investigations through the Application of a Genetic Algorithm*". Geosciences 2020, 10(7), 265; https://doi.org/10.3390/geosciences10070265 de Mello, V. F. B. 1975. "The philosophy of statistics and probability applied in soil engineering, general report." Proc. of the 2nd Int. Conf. on Application of Statistics and Probability in Soil and Structural Engineering, Aachen, 63-78.

de Mello, V. F. B. 1977. "Reflections on design decisions of practical significance to embankment dams." Geotechnique, 27(3), 281-355.

Depina, I., Le, T. M. H., Eiksund, G., and Strøm, P. 2016. "*Cone penetration data classification with Bayesian Mixture Analysis*". Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 10(1), 27-41.

Ditlevsen, O. 1981a. "*Principle of normal tail approximation.*" Journal of the Engineering Mechanics Division, 107(6), 1191-1208.

Ditlevsen, O. 1981b. "Uncertainty Modeling with Applications to Multidimensional Civil Engineering Systems". New York ; London: McGraw-Hill International Book Co.

Duncan, J.M. 2000. "Factor of safety and reliability in geotechnical engineering." Journal of Geotechnical and Geoengvironmental Engineering, 126(4), 307-316.

Duncan, J.M. and Huston, W.N. 1983. "*Estimating failure probabilities for California levees.*" Journal of Geotechnical Engineering, 109(2), 260-268.

Einstein, H. H. 1991. "Observation, Quantification, and Judgment: Terzaghi and Engineering Geology." Journal of geotechnical and Geoenvironmental Engineering, ASCE, 117(11), 1772–1778.

Einstein, H. H. 2003, 'Uncertainty in rock mechanics and rock engineering - then and now', Proceedings of the 10th ISRM Congress, International Society for Rock Mechanics and Rock Engineering, Lisbon, 281–293.

Ering, P. and Sivakumar Babu, G.L. 2016. "*Probabilistic back analysis of rainfall induced landslide- A case study of Malin landslide, India.*" Engineering Geology, 208, 154-164.

Eskesen, S. D., Tengborg, P., Kampmann, J., and Veicherts, T. H. (2004). "Guidelines for tunnelling risk management: International Tunnelling Association, Working Group No. 2." Tunnelling and Underground Space Technology, 19(3): 217-237.

Fenton, G.A. 1999a. "Estimation for stochastic soil models." Journal of Geotechnical and Geoenvironmental Engineering, 125(6), 470-485.

Fenton, G.A. 1999b. "Random field modeling of CPT data." Journal of Geotechnical and Geoenvironmental Engineering, 125(6), 486-498.

Fenton, G.A. and Griffiths, D.V. 1993. "*Statistics of block conductivity through a simple bounded stochastic medium.*" Water Resources Research, 29(6), 1825-1830.

Fenton, G.A. and Griffiths, D.V. 2003. "*Bearing capacity prediction of spatially random* $c-\varphi$ *soil.*" Canadian Geotechnical Journal, 40(1), 54-65.

Fenton, G.A. and Griffiths, D.V. 2004. "Probabilistic slope stability analysis by finite elements." Journal of Geotechnical and Geoenvironmental Engineering, 130(5), 507-518.

Fenton, G.A. and Griffiths, D.V. 2005. "*Three dimensional probabilistic foundation settlement*." Journal of Geotechnical and Geoenvironmental Engineering, 131(2), 232-239.

Fenton, G.A. and Griffiths, D.V. 2008. *Risk Assessment in Geotechnical Engineering*. Wiley.

Fenton, G.A. and Vanmarcke, E.H. 1990. "Simulation of random fields via local average subdivision." Journal of Engineering Mechanics, 116(8), 1733-1749.

Fenton, G.A., Naghibi, F., Dundas, D., Bathurst, R.J., and Griffiths, D.V. 2016. "*Reliability-based geotechnical design in 2014 Canadian highway bridge design code.*" Canadian Geotechnical Journal, 53(2), 236-251.

Folayan, J.I., Høeg, K., and Benjamin, J.R. 1970. "*Decision theory applied to settlement predictions*." Journal of the Soil Mechanics and Foundation Division, 96(SM4), 1127-1141.

Freudenthal, A. M. 1957. "Safety of Structures." Transactions, ASCE, 112, 125 - 159.

Gelman, B.A., Carlin, B.P., Stem, H.S. and Rubin, D.B. 2004. *Bayesian Data Analysis*. 2nd Ed., Chapman & Hall, London.

Ghanem, R. and Brząkała W. 1996. "*Stochastic finite-element analysis of soil layers with random interface.*" Journal of Engineering Mechanics, 122(4), 361-369.

Gilbert, R.B. 1999. *First-order Second-moment Bayesian Method for Data Analysis in Decision Making.* Geotech. Engrg. Ctr. Rep., Geotech. Engrg. Ctr., The Univ. of Texas at Austin, Austin, Texas.

Gilbert, R.B. and Tang, W.H. 1995. "*Model uncertainty in offshore geotechnical reliability*." Proceeding of Offshore Technology Conference, OTC 7757, 557-567.

Givens, G.H. and Hoeting, J.A. 2005. *Computational Statistics*, John Wiley & Sons, New York.

Goh, A.T.C., Kulhawy, F.H., and Chua, C.G. 2005. "*Bayesian neural network analysis of undrained side resistance of drilled shafts.*" Journal of Geotechnical and Geoenvironmental Engineering, 131(1), 84-93.

Gong, W., Tien, Y., Juang, C.H., Martin II, J.R., and Luo, Z. 2017. "*Optimization of site investigation program for improved statistical characterization of geotechnical property based on random field theory*". Bull. Eng. Geol. Environ., 76, 1021-1035.

Griffiths, D.V. and Fenton, G.A. 1993. "*Seepage beneath water retaining structures founded on spatially random soil.*" Geotechnique, 43(4), 577-587.

Griffiths, D.V. and Fenton, G.A. 2007. Probabilistic Methods in Geotechnical Engineering. Springer Science & Business Media.

Griffiths, D.V. and Fenton, G.A., 2001. "*Bearing capacity of spatially random soil: The undrained clay Prandtl problem revisited.*" Géotechnique, 54(4), 351-359.

Griffiths, D.V., Huang, J., and Fenton, G.A. 2009a. "Influence of spatial variability on slope reliability using 2-D random fields." Journal of Geotechnical and Geoenvironmental Engineering, 135(10), 1367-1378.

Griffiths, D.V., Huang, J., and Fenton, G.A. 2009b. On the reliability of earth slopes in three dimensions. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 465(2110): 3145-3164.

Griffiths, D.V., Huang, J., and Fenton, G.A. 2011. "Probabilistic infinite slope analysis." Computers and Geotechnics, 38(4), 577-584.

Hadjigeorgiou, J. 2019. “Understanding, managing and communicating geomechanical mining risk”. Proc., First International Conference on Mining Geomechanical Risk, Perth, Australia, 3-20, https://doi.org/10.36487/ACG_rep/1905_0.1_Hadjigeorgiou

Haldar, A. and Tang, W.H. 1979a. “*Uncertainty analysis of relative density.*” Journal of Geotechnical Engineering Division, 105(GT7), 899-904.

Haldar, A. and Tang, W.H., 1979b. “*Probabilistic evaluation of liquefaction potential.*” Journal of the Geotechnical Engineering Division, 105(2), 145-163

Hammitt, G.M. 1966. “*Statistical analysis of data from comparative laboratory test program sponsored by ACIL*”, Miscellaneous Paper 4-785, U.S. Army Engineer Waterways Experiment Station, Vicksburg, 1966, 40 p.

Hansen, J.B. 1956. “Brudstadieberegning og partialsikkerheder i geoteknikken. (Limit design and safety factors in soil mechanics.)” Ingenieren 65, No. 18, 383-385.

Hansen, J.B. 1965. “*Philosophy of foundation design: design criteria, safety factors and settlement limits.*” Proceedings, Symposium on Bearing Capacity and Settlement of Foundations, Duke University, Durham, 1-13.

Haque, M. and Abu-Farsakh, M. 2018. “*Estimation of pile setup and incorporation of resistance factor in load resistance factor design framework*.” Journal of Geotechnical and Geoenvironmental Engineering, 144(11), 04018077.

Harr, M. E. 1977. Mechanics of Particulate Media : A Probabilistic Approach. New York: McGraw-Hill.

Harr, M. E. 1987. Reliability-Based Design in Civil Engineering. New York: McGraw-Hill Book Company.

Hartford, D. and Baecher, G.B. 2004. *Risk and Uncertainty in Dam Safety*, Thomas Telford, Ltd., London, 416pp., 2004

Hasofer, A.M. and Lind, M.C. 1974. “*An exact and invariant first order reliability format*.” Journal of Engineering Mechanics, 100, 111-121.

He, L., Liu, Y., Bi, S., Wang, L., Broggi, M., and Beer, M. 2020. “*Estimation of failure probability in braced excavation using Bayesian networks with integrated model updating*.” Underground Space, 5(4), 315-323.

Hicks, M.A. and Samy, K. 2002. “*Reliability-based characteristic values: a stochastic approach to Eurocode 7*.” Ground Engineering, 35(12), 30-34.

Hicks, M.A. and Spencer, W.A. 2010. “*Influence of heterogeneity on the reliability and failure of a long 3D slope.*” Computers and Geotechnics, 37(7-8), 948-955.

Hicks, M.A., Chen, J., and Spencer, W.A. 2008. “*Influence of spatial variability on 3D slope failures.*” Proc. 6th Int. Conf. on Computer Simulation in Risk Analysis and Hazard Mitigation, 335-342.

Høeg, K. 1996. “*Performance evaluation, safety assessment and risk analysis for dams*.” Hydropower and Dams, Issue 6, Nov. 1996, 8 p.

Høeg, K. and Murarka, R.P. 1974. “*Probabilistic analysis and design of a retaining wall.*” Journal of the Geotechnical Engineering Division, 100(GT3), 349-366.

Hoek, E. 1999. “Putting numbers to geology—an engineer’s viewpoint.” Quarterly Journal of Engineering Geology and Hydrogeology, 32(1), 1–19.

Hohenbichler, M. and Rackwitz, R., 1981. “*Non-normal dependent vectors in structural safety.*” Journal of the Engineering Mechanics Division, 107(6), 1227-1237.

Honjo, Y. 2005. “*JGS comprehensive foundation design code: Geocode 21*.” Proc. 16th ICSMGE, Osaka, 2813-2816.

Honjo, Y. 2011. “*Challenges in geotechnical reliability based design*.” ISGSR 2011 - Vogt, Schuppener, Straub & Bräu (eds), Bundesanstalt für Wasserbau ISBN 978-3-939230-01-4.

Honjo, Y. and Kashiwagi, N. 1999. “*Matching objective and subjective information in groundwater inverse analysis by Akaike Bayesian information criterion*.” Water Resour. Res., 35(2), 435-447.

Honjo, Y., Kikuchi, Y., and Shirato, M. 2010. “*Development of the design codes grounded on the performance based design concept in Japan*.” Soils and Foundations, 50(6), 983-1000.

Honjo, Y., Liu, W.T., and Soumitra, G. 1994. “*Inverse analysis of an embankment on soft clay by extended Bayesian method*.” Int. J. Numer. Anal. Methods Geomech, 18, 709-734.

Hu, J.Z., Zhang, J., Huang, H.W., and Zheng, J.G., 2021. “*Value of information analysis of site investigation program for slope design*.“ Computers and Geotechnics, 131, 103938.

Huang H.W. (2021). Technical Specification for Infrastructure Monitoring by Wireless Sensor Network (in Chinese) (T/CCES 17-2020), China Civil Engineering Society Code.

Huang, H. W. and Zhang, D. M. (2016). “Resilience analysis of shield tunnel lining under extreme surcharge: Characterization and field application.” Tunnelling and Underground Space Technology, 51: 301-312.

Huang, H. W., Zhang, D. M., and Ayyub, B. M. 2017. “An integrated risk sensing system for geo-structural safety.” Journal of Rock Mechanics and Geotechnical Engineering, 9(2): 226-238.

Huang, H., Shi, J., Wang, F., Zhang, D., and Zhang, D. 2020. “Theoretical and experimental studies on the signal propagation in soil for wireless underground sensor networks.” Sensors, 20(9): 2580.

Huang, H.W., S. C. Wen, J. Zhang, F. Y. Chen, J. R. Martin, and H. Wang. 2018. “*Reliability analysis of slope stability under seismic condition during a given exposure time*.” Landslides, 15 (11): 2303–2313.

Huang, J., Fenton, G.A., Griffiths, D.V., Li, D., and Zhou, C. 2017. “On the efficient estimation of small failure probability in slopes.” *Landslides*, 14(2), 491-498.

Huang, J., Griffiths, D.V., and Fenton, G.A. 2010. System reliability of slopes by RFEM. *Soils and Foundations*, 50(3), 343-353.

Huang, J., Lyamin, A.V., Griffiths, D.V. Krabbenhoft, K., and Sloan, S.W. 2013. Quantitative risk assessment of landslide by limit analysis and random fields. *Computers and Geotechnics*, 53, 60-67.

Huang, J., Zheng, D., Li, D., Kelly, R., and Sloan, S. 2018. "Probabilistic characterization of 2D soil profile by integrating CPT with MASW data." *Canadian Geotechnical Journal*, 55(8): 1168-1181

International Organization for Standardization. 2015. *General Principles on Reliability for Structures*. ISO2394:1973/1986/1998/2015. Geneva, Switzerland.

Ishii, K. and Suzuki, M. 1986. “*Stochastic finite element method for slope stability analysis.*” Structural Safety, 4, 111-129.

Jahanfar, A., Gharabaghi, B., McBean, E.A., and Dubey, B.K. 2016. “*Municipal solid waste slope stability modeling: a probabilistic approach.*” J. Geotech. Geoenviron. Eng., 143(8), 04017035.

Jaksa, M.B. 1995. *The Influence of Spatial Variability on the Geotechnical Design Properties of a Stiff, Overconsolidated Clay.* Ph.D. thesis, Univ. of Adelaide, Australia.

Jaksa, M.B., Goldsworthy, J.S., Fenton, G.A., Kaggwa, W.S., Griffiths, D.V., Kuo, Y.L., and Poulos, H.G. 2005. Towards reliable and effective site investigations. Géotechnique, 55(2), 109-121

Jaksa, M.B., Kaggwa, W.S., and Brooker, P.I. 1999. “*Experimental evaluation of the scale of fluctuation of a stiff clay*.” Proc. of the 8th Int. Conf. on Application of Statistics and Probability, AA Balkema, Rotterdam, 415-422.

Jaksa, M.B., Maier, H.R., and Shahin, M.A. 2008. “*Future challenges for artificial neural network modelling in geotechnical engineering.*” The 12th international conference of international association for computer methods and advances in geomechanics (IACMAG), Geo, India, 1710-1719.

Javankhoshdel, S., Luo, N., and Bathurst, R.J. 2017. “Probabilistic analysis of simple slopes with cohesive soil strength using RLEM and RFEM.” Georisk, 11(3), 231-246.

JGS. 2006. *Principles for Foundation Designs Grounded on a Performance Based Design Concept*. JGS4001-2004, Japanese Geotechnical Society.

Jiang, S.H., Papaioannou, I., and Straub, D. 2020. “*Optimization of site-exploration programs for slope-reliability assessment*” ASCE-ASME J. Risk Uncertainty Eng. Syst. Part A: Civ. Eng., 6(1), 0001042.

Johnston, M.M. 1969. “*Laboratory comparison tests using compacted fine-grained soils*.”, Proceedings, Seventh International Conference on Soil Mechanics & Foundation Engineering (1), Mexico City, 197-202.

Jong, S. C., Ong, D. E. L., and Oh, E. 2021. State-of-the-art review of geotechnical-driven artificial intelligence techniques in underground soil-structure interaction. Tunnelling and Underground Space Technology, 113, 103946.

Juang, C.H. and Wang, L. 2013. “Reliability-based robust geotechnical design of spread foundations using multi-objective genetic algorithm.” Computers and Geotechnics, 48, 96-10.

Juang, C.H. and Zhang, J. 2017. “Bayesian methods for geotechnical applications - A practical guide.” In Geotechnical Safety and Reliability: Honoring Wilson H. Tang (GSP 286), ASCE, Reston.

Juang, C.H., Gilbert, R.B., Zhang, L.M., Zhang, J., and Zhang, L. 2017. *Geotechnical Safety and Reliability: Honoring Wilson H. Tang*. ASCE GSP 286. Reston.

Juang, C.H., Jiang, T., and Andrus, R. 2002. “*Assessing probability-based methods for liquefaction potential evaluation.*” Journal of Geotechnical and Geoenvironmental Engineering, 128(7), 580-589.

Juang, C.H., Luo, Z., Atamturktur, S., and Huang, H. 2013b. “*Bayesian updating of soil parameters for braced excavations using field observations.*” Journal of Geotechnical and Geoenvironmental Engineering, 139(3), 395-406.

Juang, C.H., Wang, L., Liu, Z., Ravichandran, N., Huang, H.W., and Zhang, J. 2013a. “Robust geotechnical design of drilled shafts in sand: New design perspective.” Journal of Geotechnical and Geoenvironmental Engineering, 139(12), 2007-2019.

Juang, C.H., Zhang, J., Shen, M.F., and Hu, J.Z. 2019. "*Probabilistic methods for unified treatment of geotechnical and geological uncertainties in a geotechnical analysis*." Engineering Geology, 249, 148-161.

Kalmogo, P., Sritharan, S., and Ashlock, J.C. 2019. *Recommended Resistance Factors for Load and Resistance Factor Design of Drilled Shafts in Iowa.* Report No. InTrans Project 14-512. Ames, IA: Iowa Department of Transportation.

Kang, F., Xu, Q., and Li, J. 2016. "*Slope reliability analysis using surrogate models via new support vector machines with swarm intelligence.*" Applied Mathematical Modelling, 40(11-12), 6105-6120.

Kawa, M., Puła, W., Truty, A. 2021. "*Probabilistic analysis of the diaphragm wall using the hardening soil-small (HSs) model*". Eng. Struct., 232 (2021), p. 111869

Kay, J.N. 1976. "*Safety factor evaluation for single piles in sand*." J. Geotech. Eng. Div., 102(10), 1093-1108.

Kay, J.N. and Krizek, R.J. 1971. "*Estimation of mean for soil properties.*" Proceedings, First International Conference on Applications of Statistics & Probability to Soil & Structural Engineering, Hong Kong, 279-286.

Kitanidis, P. K. 1997. Introduction to geostatistics : applications to hydrogeology. Cambridge University Press, New York.

Kraft, Jr., L. M., and Murff, J. D. 1976. "Probabilistic Investigation of Foundation Design for Offshore Gravity Structures." Society of Petroleum Engineers Journal, 16(2), 13.

Kraft, Jr., L.M. 1968. *Probability Applications in Soils Engineering*. Ph.D., The Ohio State University, United States.

Kulhawy, F.W., Roth, N.J., and Grigoriu, M.D. 1991. "*Some statistical evaluations of geotechnical properties.*" Proc. of the 6th Int. Conf. of Statistics and Probability in Soil and Structural Eng. (ICASP), Mexico City, 705-712.

Lacasse, S. 2015. "*Hazard, risk and reliability in geotechnical practice*." 55th Rankine Lecture, Institution of Civil Engineers, London, UK.

Lacasse, S. 2016. "*Hazard, reliability and risk assessment - research and practice for increased safety*." Keynote Paper, Proceedings of the 17th Nordic Geotechnical Meeting NGM 2016, Reykjavik.

Lacasse, S. and Goulois, A. 1989. "*Uncertainty in API parameters for predictions of axial capacity of driven pile in sand.*" Proc. 21st OTC Houston, Texas, USA, Paper 6001, pp. 353- 358.

Lacasse, S. and Lamballerie, J.-Y.N. 1995. "*Statistical treatment of cone penetration testing*." Intern. Symp. on Cone Penetration Testing CPT '95, Linköping, Sweden. 2, 369-377.

Lacasse, S. and Nadim, F. 1994. "*Reliability issues and future challenges in geotechnical engineering for offshore structures.*" Plenum Paper, 7th International Conference on Behaviour of Offshore Structures BOSS'94. MIT Press, Cambridge, MA, USA.

Lacasse, S. and Nadim, F. 1996a. "*Uncertainties in characterising soil properties*." In: Uncertainty in the geologic environment: From Theory to Practice, Ed. C.D. Shackelford et al., ASCE, Reston, 49-75.

Lacasse, S. and Nadim, F. 1996b. "*Model uncertainty in pile axial capacity calculations*." Proc. 28th OTC Houston, TX, USA, paper 7996, 369-380.

Lacasse, S. and Nadim, F. 2007. "*Probabilistic geotechnical analyses for offshore facilities*." Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 1(1), 21-42.

Lacasse, S., Liu, Z., and Nadim, F. 2017. "*Probabilistic characterization of soil properties — Recognition of Wilson Tang's contribution to geotechnical practice*." Geo-Risk 2017. https://ascelibrary.org/doi/10.1061/9780784480731.002

Lacasse, S., Nadim, F., Liu, Z.Q., Eidsvig, U.K., Le, T.M.H., and Lin, C.G. 2019. "*Risk assessment and dams – Recent developments and applications.*" Keynote Paper, Proceedings of the XVII ECSMGE-2019 Geotechnical Engineering foundation of the future, Reykjavik, doi: 10.32075/17ECSMGE-2019-1110.

Langejan, A. 1965. "*Some aspects of the safety factor in soil mechanics and foundation engineering*." Proc. of 6th Int. Conf. on Soil Mechanics and Foundation Engineering, Montreal, Vol II, 500-502.

Ledesma, A., Gens A., and Alonso E.E. 1996a. "*Estimation of parameters in geotechnical backanalysis-I. Maximum likelihood approach*." Comput. Geotech., 18 (1), 1-27.

Ledesma, A., Gens, A., and Alonso, E.E. 1996b. "*Parameter and variance estimation in geotechnical back analysis using prior information*." Int. J. Numer. Anal. Methods Geomech., 20, 114-141.

Levi, R. 1958. "*Les calculus de securote en matière de fondations.*" Annales de l'Institut Technique du Batiment et des Travaux Publics., 11, 541-544.

Li, D.Q., Zheng, D., Cao, Z. J., Tang, X. S. & Phoon, K. K. 2016b. "Response Surface Methods for Slope Reliability Analysis: Review and Comparison", Engineering Geology, 203, 3-14.

Li, D.Q., Chen, Y., Lu, W., and Zhou, C. 2011. "*Stochastic response surface method for reliability analysis of rock slopes involving correlated non-normal variables*". Computers and Geotechnics, 38(1), 58-68.

Li, D.Q., Jiang, S.H., Cao, Z.J., Zhou, C.B., and Zhang, L.M. 2015. "*A multiple response-surface method for slope reliability analysis considering spatial variability of soil properties*". Engineering Geology, 187, 60-72.

Li, D.Q., Qi, X.H., Cao, Z.J., Tang, X.S., Phoon, K.K., and Zhou C.B. 2016a. "*Evaluating slope stability uncertainty using coupled Markov chain*". Computers and Geotechnics, 73, 72-82.

Li, D.Q., Xiao, T., Cao, Z. J., Zhou, C. B. and Zhang, L. M. 2016c. "*Enhancement of random finite element method in reliability analysis and risk assessment of soil slopes using Subset Simulation*". Landslides, 13, 293–303.

Li, J., Hu, P., Uzielli, M., and Cassidy, M.J. 2018. "*Bayesian prediction of peak resistance of a spudcan penetrating sand-over-clay*". Géotechnique, 68(10), 905-917.

Li, K.S. and Lumb, P. 1987. "*Probabilistic design of slopes.*" Canadian Geotechnical Journal, 24(4), 520-535.

Li, K.S., Lee, I.K., and Lo, S-C.R. 1993. "*Limit state design in geotechnics*." Proc. of the Conf. on Probabilistic Methods in Geotechnical Engineering, Canberra, Eds. K.S. Li and S-C. R. Lo, Balkema, Rotterdam, 29-42.

Li, L., Wang, Y. and Cao, Z.J. 2013. "*Risk de-aggregation and system reliability analysis of slope stability using representative slip surfaces*". Computers and Geotechnics, 53, 95-105.

Li, Y., Fenton, G.A., Hicks, M.A., and Xu, N. 2021. “*Probabilistic bearing capacity prediction of square footings on 3D spatially varying cohesive soils.*” Journal of Geotechnical and Geoenvironmental Engineering, 147(6), 04021035.

Li, Z., Huang, H.W., Nadim, F., and Xue, Y. 2010. “Quantitative risk assessment of cut-slope projects under construction.” Journal of Geotechnical and Geoenvironmental Engineering, 136(12): 1644-1654.

Lin, P. and Bathurst, R.J. 2019. “*Calibration of resistance factors for load and resistance factor design of internal limit states of soil nail walls.*” Journal of Geotechnical and Geoenvironmental Engineering, 145(1), 04018100

Lloret-Cabot, M.F.G.A., Fenton, G.A., and Hicks, M.A. 2014. “*On the estimation of scale of fluctuation in geostatistics.*” Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 8(2), 129-140.

Lo, S-C.R., Li, K.S., and Lee, I.K. 1992. “*Limit state design of pile foundations: a probabilistic appraisal*.” Proc. of 6th Australian-New Zealand Conf. on Geomechanics, Christchurch.

Low, B. K. 2021. Reliability-Based Design in Soil and Rock Engineering: Enhancing Partial Factor Design Approaches. CRC Press.

Low, B.K. and Tang, W.H. 1997. “*Efficient reliability evaluation using spreadsheet*.” Journal of Engineering Mechanics, 123(7), 749-752.

Low, B.K. and Tang, W.H. 2007. “*Efficient spreadsheet algorithm for first order reliability method.*” Journal of Engineering Mechanics, 133(12), 1378-1387.

Low, B.K., Gilbert, R.B., and Wright, S.G. 1998. “Slope reliability analysis using generalized method of slices.” Journal of Geotechnical and Geoenvironmental Engineering, 124(4), 350-362.

Low, B.K., Lacasse, S. and Nadim, F. 2007. “*Slope reliability analysis accounting for spatial variation*.” Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 1(4), 177-189.

Lumb, P. 1966. “*The variability of natural soils.*” Canadian Geotechnical Journal, 3(2), 74-97.

Lumb, P. 1970. “*Safety factors and the probability distributions of soil strength.*” Canadian Geotechnical Journal, 7(3), 225-242.

Lumb, P. 1974. “*Application of statistics in soil mechanics*.” In: Soil Mechanics – New Horizons, Ed. I. K. Lee, Newness-Butterworths, London, 44-111.

Luo, N. and Bathurst, R.J. 2018a. “*Deterministic and random FEM analysis of full-scale unreinforced and reinforced embankments.*” Geosynthetics International, 25(2), 164-179.

Luo, N. and Bathurst, R.J. 2018b. “*Probabilistic analysis of reinforced slopes using RFEM and considering spatial variability of frictional soil properties due to compaction*.” Georisk, 12(2), 87-108.

Luo, Z., Li, Y., Zhou, S., and Di, H. 2018. “Effects of vertical spatial variability on supported excavations in sands considering multiple geotechnical and structural failure modes.” Comput Geotech, 95, 16-29.

Macedo, J., J. Bray, N. Abrahamson, and T. Travasarou. 2018. “*Performance-Based Probabilistic Seismic Slope Displacement Procedure*.” Earthquake Spectra, 34(2), 673–695.

McAnally, P.A. 1983. “*Reliability of the bearing capacity design of shallow footings in sands.*” Proc. 4th Int. Conf. on Appl. Of Statistics and Probability in Soil and Structural Eng. (ICASP), Florence, 1545-1556.

McVay, M.C., Birgisson, B., Zhang, L.M., Perez, A., and Putcha, S. 2000. "*Load and resistance factor design (LRFD) for driven piles using dynamic methods-a Florida perspective*." Geotechnical Testing Journal, 23(1), 55-66.

McVay, M.C., Kuo, C.L., and Singletary, W.A. 1998. *Calibrating Resistance Factors in the Load and Resistance Factor Design for Florida Foundations*. Tallahassee, FL: Florida Department of Transportation.

Mitchell, J. K. and Kopmann, J. 2013. The Future of Geotechnical Engineering. Report CGPR #70. Virginia Tech Center for Geotechnical Practice and Research.

Moss, R.E.S., Seed, R.B., Kayen, R.E., Steward, J.P., and Der Kiureghian, A. 2006. "*CPT-based probabilistic and deterministic assessment of in situ seismic soil liquefaction potential*." J. Geotech. Geoenviron. Eng., 132(8), 1302-1051.

Motamed, R., Elfass, S., and Stanton, K. 2016. *LRFD Resistance Factor Calibration for Axially Loaded Drilled Shafts in the Las Vegas Valley.* Report No. 515-13-803. Reno, NV: Nevada Department of Transportation.

Nadim, F. 2017. "*Reliability-based approach for robust geotechnical design*." Third Suzanne Lacasse Honour Lecture, ISSMGE TC304. Proc. 19th International Conference on Soil Mechanics and Geotechnical Engineering, Seoul, South Korea.

Nadim, F. and Gudmestad, O.T. 1994. "*Reliability of an engineering system under a strong earthquake with application to offshore platforms*." Structural Safety, 4, 151-163.

Nadim, F. and Kvalstad, T.J. 2007. "*Risk assessment and management for offshore geohazards*." Proc. First International Symposium on Geotechnical Safety and Risk ISGSR 2007, Shanghai, PRC.

Nadim, F. and Lacasse, S. 1992. "*Probabilistic bearing capacity analysis of jack-up structures*." Can. Geotech. J., 29, 580-588.

Najjar, S.S. and Gilbert, R.B. 2009. "Importance of lower-bound capacities in the design of deep foundations." Journal of Geotechnical and Geoenvironmental Engineering, 135(7), 890-900.

National Research Council. 1995. Probabilistic Methods in Geotechnical Engineering. National Academies Press, Washington, DC.

Ng, K.W., Adhikari, P., and Gebreslasie, Y.Z., 2019. *Development of Load and Resistance Factor Design Procedures for Driven Piles on Soft Rocks in Wyoming*. Report No. WY-1902F. Cheyenne, WY: Wyoming Department of Transportation.

Ng, T. and Fazia, S. 2012. *Development and Validation of a Unified Equation for Drilled Shaft Foundation Design in New Mexico*. Report No. NM10MSC-01. Albuquerque, NM: New Mexico Department of Transportation.

Nowak, A.S., Kozikowski, M., Larsen, J., Lutomirski, T., and Paczkowski, P. 2007. *Implementation of the AASHTO LRFD Code in Geotechnical Design-Piles.* Report No. SPR-1(07) P595. Lincoln, NE: Nebraska Department of Roads.

Orr, T.L.L. 2015. Managing risk and achieving reliable geotechnical designs using Eurocode 7. Chapter 10 in Risk and Reliability in Geotechnical Engineering. Edited by Phoon, K. K. & Ching, J. CRC Press.

Orr, T.L.L. 2000. “*Selection of characteristic values and partial factors in geotechnical design to Eurocode 7*.” Computers and Geotechnics, 25(3-4), 263-279.

Orr, T.L.L. and Farrell E.R. 1999. *Geotechnical Design to Eurocode 7*, Springer Verlag, London.

Paice, G.M., Griffiths, D.V., and Fenton, G.A. 1996. “*Finite element modelling of settlements on spatially random soil.*” Journal of Geotechnical Engineering, 122(9), 777-779.

Paikowsky, S.G., Birgisson, B., McVay, M., Nguyen, T., Kuo, C., Baecher, G.B., Ayyub, B., Stenersen, K., O'Malley, K., Chernauskas, L., and O'Neill, M. 2004. *Load and Resistance Factor Design (LRFD) for Deep Foundations.* NCHRP Report 507, Washington, DC: Transportation Research Board of the National Academies.

Paikowsky, S.G., Canniff, M. C., Lesny, K., Kisse, A., Amatya, S., and Muganga, R. 2010. *LRFD Design and Construction of Shallow Foundations for Highway Bridge Structures*. NCHRP Report 651. Washington, DC: Transportation Research Board of the National Academies.

Penfield, J., Parsons, R., Han, J., and Misra, A. 2014. *Load and Resistance Factor Design Calibration to Determine a Resistance Factor for the Modification of the Kansas Department of Transportation – Engineering News Record Formula.* Report No. K-TRAN: KU-13-4. Topeka, KA: Kansas Department of Transportation.

Petek, K., McVay, M., and Mitchell, R. 2020. *Development of Guidelines for Bearing Resistance of Large Diameter Open-end Steel Piles.* Report No. FHWA-HRT-20-011. McLean, VA: U.S. Department of Transportation and Federal Highway Administration.

Phoon K.K. and Kulhawy F.H. 1999a. “*Characterizations of geotechnical variability*.” Canadian Geotechnical Journal, 36(4), 612-624.

Phoon, K.K. 2008. Reliability-Based Design in Geotechnical Engineering: Computations and Applications, Taylor & Francis, 532 p.

Phoon, K. K. and Retief, J. V. 2016. Reliability of Geotechnical Structures in ISO2394, CRC Press/Balkema, 230 p.

Phoon, K. K. and Zhang, W. (2022). Future of Machine Learning in Geotechnics. Georisk: Assessment & Management of Risk for Engineered Systems & Geohazards, under review.

Phoon, K. K., Cao, Z., Liu, Z. and Ching, J. 2022b. Report for ISSMGE TC309/TC304/TC222 Third ML dialogue on “Data-Driven Site Characterization (DDSC).” Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, under review.

Phoon, K. K., Ching, J., and Wang Y. 2019. *“Managing risk in geotechnical engineering – from data to digitalization”*. Proceedings, 7th International Symposium on Geotechnical Safety and Risk (ISGSR 2019), Taipei, Taiwan, 13-34.

Phoon, K. K., Quek, S. T., Chow, Y. K., and Lee, S. L. 1990. "Reliability analysis of pile settlement.” Journal of Geotechnical Engineering, ASCE, 116(11), 1717 - 1735.

Phoon, K.K. 2017. “*Role of reliability calculations in geotechnical design*.” Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 11(1), 4-21. https://doi.org/10.1080/17499518.2016.1265653

Phoon, K.K. 2020. “*The Story of statistics in geotechnical engineering*.” Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 14(1), 3-25. https://doi.org/10.1080/17499518.2019.1700423

Phoon, K.K. and Ching, J. 2016. “*Semi-probabilistic reliability-based design*.” Chapter 6, Reliability of Geotechnical Structures in ISO2394, CRC Press/Balkema, 159-192.

Phoon, K.K. and Ching, J. 2021. “*Project DeepGeo – data-driven 3D subsurface mapping*.” Journal of GeoEngineering, 16(2), 61-74.

Phoon, K.K. and Kulhawy F.H. 1999b. “*Evaluation of geotechnical property variability.*” Canadian Geotechnical Journal, 36(4), 625-639.

Phoon, K.K. and Tang, C. 2019. “*Characterization of geotechnical model uncertainty*.” Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 13(2), 101-130.

Phoon, K.K., Ching, J. and Cao, Z. 2022a. Unpacking data-centric geotechnics. Underground Space, in press.

Phoon, K.K., Ching, J., and Shuku, T. 2021. “*Challenges in data-driven site characterization.*” Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, https://doi.org/10.1080/17499518.2021.1896005

Phoon, K.K., Kulhawy, F.H., and Grigoriu, M. D. 1995. *Reliability-based Design of Foundations for Transmission Line Structures.* ERPI report TR-105000, Palo Altos, CA, USA.

Phoon, K.K., Kulhawy, F.H., and Grigoriu, M.D. 2003a. “*Development of a reliability-based design framework for transmission line structure foundations*.” Journal of Geotechnical and Geoenvironmental Engineering, 129(9), 798-806.

Phoon, K.K., Kulhawy, F.H., and Grigoriu, M.D. 2003b. “*Multiple resistance factor design (MRFD) for spread foundations*”. Journal of Geotechnical and Geoenvironmental Engineering, 129(9), 807-818.

Phoon, K.K., Lacasse, S., Fenton, G.A., Honjo, Y., and Nadim, F. 2007. Editorial. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 1(1), 1-2.

Phoon, K.K., Prakoso, W.A., Wang, Y., and Ching, J. 2016. “*Uncertainty representation of geotechnical design parameters*”. Chapter 3, Reliability of Geotechnical Structures in ISO2394, CRC Press/Balkema, 49-87.

Pieczyńska-Kozłowska, J.M., Puła, W., and Vessia, G. 2017. “*A collection of fluctuation scale values and autocorrelation functions of fine deposits in Emilia Romagna plain (Italy)*.” Geo-Risk 2017 in ASCE Geotechnical Special Publication 284, 290-299.

Priest, S. and Hudson, J. 1981. “Estimation of discontinuity spacing and trace length using scanline surveys”. International Journal of Rock Mechanics and Mining Sciences, 18(3), 183–197.

Przewłócki, J. 2005. “*A stochastic approach to the problem of bearing capacity by the method of characteristics.*” Computers and Geotechnics, 32(5), 370-376.

Pugsley, A. (1955). "Report on Structural Safety." Structural Engineer, 33(5), 141 - 149.

Quek, S. T., Chow, Y. K., and Phoon, K. K. 1992. "Further contributions to reliability-based pile settlement analysis." Journal of Geotechnical Engineering, ASCE, 118(5), 726 - 742.

Quek, S. T., Phoon, K. K., and Chow, Y. K. 1991. "Pile group settlement: a probabilistic approach." International Journal for Numerical & Analytical Methods in Geomechanics, 15(11), 817 - 832.

Rackwitz R. and Flessler, B. 1978. "*Structural reliability under combined random loads sequences*." Computers and Structures, 9, 489-494.

Rahman, M.S., Gabr, M.A., Sarcia, R.Z., and Hossain, M.S. 2002. *Load and Resistance Factor Design (LRFD) for Analysis/Design of Piles Axial Capacity*. Report No. FHWA/NC/2005-08. Raleigh, NC: North Carolina Department of Transportation.

Reddi, L.N. and Wu, T.H. 1991. "*Probabilistic analysis of ground-water levels in hillside slopes*." J. Geotech. Eng., 117(6), 872-890.

Righetti, G. and Harrop-Williams, K. 1988. "*Finite element analysis of random soil media.*" Journal of Geotechnical Engineering, 11(1), 59-75.

Salgado, R., Woo, S.I., and Kim, D. 2011. *Development of Load and Resistance Factor Design for Ultimate and Serviceability Limit States of Transportation Structure Foundations*. Report No. FHWA/IN/JTRP-2011/03. Indianapolis, IN: Indiana Department of Transportation.

Savvides, A.A., Papadrakakis, M. 2021. "A computational study on the uncertainty quantification of failure of clays with a modified Cam-Clay yield criterion." SN Appl. Sci. 3, 659. https://doi.org/10.1007/s42452-021-04631-3

Schultze, E. 1972. "*Frequency distributions and correlations of soil properties*." In: Proc. of the First Int. Conf. of Statistics and Probability in Soil and Structural Engineering (ICASP), Ed. P. Lumb, Hong-Kong University Press, Hong-Kong, 372-387.

Schultze, E. 1975. "*Some aspects concerning the application of statistics and probability to foundation structures*." In: Proc. of the 2nd Int. Conf. of Statistics and Probability in Soil and Structural Engineering (ICASP), Aachen, 457-494.

Seo, H., Moghaddam, R.B., Surles, J.G., and Lawson, W.D. 2015. *Implementation of LRFD Geotechnical Design for Deep Foundations using Texas Cone Penetrometer (TCP) Test*. Report No. FHWA/TX-16/6-6788-01-1. Austin, TX: Texas Department of Transportation.

Sert, S., Luo, Z., Xiao, J., Gong, W., Juang, C.H. 2016. "*Probabilistic analysis of responses of cantilever wall-supported excavations in sands considering vertical spatial variability*". Comput Geotech, 75 (2016), pp. 182-191

Shackelford, C.D., Nelson, P.P., and Roth, M.J.S. 1996. "*Uncertainty in the geologic environment: from theory to practice*." Geotechnical Special Publications 58, ASCE, New York.

Shahin, M.A., Jaksa, M.B., and Maier, H.R. 2001. "*Artificial neural network applications in geotechnical engineering.*" Australian geomechanics, 36(1), 49-62.

Shahin, M.A., Jaksa, M.B., and Maier, H.R. 2008. "*State of the art of artificial neural networks in geotechnical engineering.*" Electronic Journal of Geotechnical Engineering, 8(1), 1-26.

Shi, C. and Wang, Y. (2021). Development of subsurface geological cross-section from limited site-specific boreholes and prior geological knowledge using iterative convolution XGBoost. Journal of Geotechnical and Geoenvironmental Engineering, 147(9), 04021082.

Shuku, T., Phoon, K. K., and Yoshida, I. 2020. “*Trend estimation and layer boundary detection in depth-dependent soil data using sparse Bayesian lasso*.” Computers and Geotechnics, 128, 103845.

Singh, A. 1972. “*How reliable is the safety factor in foundation engineering*?” In: Proc. of the First Int. Conf. of Statistics and Probability in Soil and Structural Engineering (ICASP), Ed. P. Lumb, Hong-Kong University Press, Hong-Kong, 389-424.

Singh, A. and Lee, K.L. 1970. “*Variability in soil parameters*”, Proceedings, Eighth Annual Engineering Geology & Soils Engineering Symposium, Idaho, 159-185.

Smith, L. and Freeze, R.A. 1979a. “*Stochastic analysis of steady state groundwater flow in a bounded domain: 1. One-dimensional simulations*.” Water Resources Research, 15(3), 521-528.

Smith, L. and Freeze, R.A. 1979b. “*Stochastic analysis of steady state groundwater flow in a bounded domain: 2. Two-dimensional simulations*.” Water Resources Research, 15(6), 1543-1559.

Smith, T., Banas, A., Gummer, M., and Jin, J. 2011. *Recalibration of the GRLWEAP LRFD Resistance Factor for Oregon DOT*. Report No. FHWA-OR-RD-11-08. Salem, OR: Oregon Department of Transportation.

Spanos, P.D. and Ghanem, R. 1989. “Stochastic finite element expansion for random media”. Journal of Engineering Mechanics, 115(5), 1035-1053.

Tabarroki, M., Ching, J., Phoon, K.K., and Chen, Y.Z. 2020. “*Mobilisation-based characteristic value of shear strength for ultimate limit states*.” Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, https://doi.org/10.1080/17499518.2020.1859121.

Tang, C. and Phoon, K.K. 2018. “*Statistics of model factors and consideration in reliability-based design of axially loaded helical piles*.” Journal of Geotechnical and Geoenvironmental Engineering, 144(8), 04018050.

Tang, C. and Phoon, K.K. 2021. *Model Uncertainties for Foundation Design*, CRC Press.

Tang, W.H. 1971. "*A Bayesian evaluation of information for foundation engineering design.*” In Statistics and Probability in Civil Engineering, Proceedings, 1st Intl. Conference on Applications of Statistics and Probability to Soil and Structural Engineering, Hong Kong University Press, 173-85.

Tang, W.H. 1973. "*Modeling, analysis and updating of uncertainties.*” ASCE National Meeting on Structural Engineering, San Francisco; Preprint No. 2016.

Tang, W.H. 1984. "*Principles of probabilistic characterization of soil properties.*" Probabilistic Characterization of Soil Properties: Bridge between Theory and Practice, ASCE, 74-89.

Tang, W.H. 1987. "*Updating anomaly statistics - single anomaly case*.” Structural Safety, 4, 151-163.

Tang, W.H. and Halim, I. 1988. "*Updating anomaly statistics - multiple anomaly pieces.*” Journal of Engineering Mechanics, 114(6), 1091-1096.

Tang, W.H. and Quek, S.T. 1986. "*Statistical model of boulder size and fraction*.” Journal of Geotechnical Engineering, 112(1), 79-90.

Tang, W.H., Halim, I., and Gilbert, R.B. 1988. "*Reliability of geotechnical systems considering geologic anomaly*.” Proceedings, ASCE Conference on Probabilistic Methods, VPI, 136-139

Tang, W.H., Youcemen, M.S., and Ang, A.H.S. 1976. “*Probability based short-term design of soil slopes*.” Canadian Geotechnical Journal, 13(3), 201-215.

Taylor, D. W. (1948). Fundamentals of Soil Mechanics. New York: John Wiley & Sons.

TC304. 2021. *304dB – TC304 Databases*. ISSMGE TC304 (Technical Committee on Engineering Practice of Risk Assessment and Management), International Society for Soil Mechanics and Geotechnical Engineering. Accessed September 14, 2021. http://140.112.12.21/issmge/tc304.htm?=6

Uzielli, M. and Mayne, P.W. 2011. “Serviceability limit state CPT-based design for vertically loaded shallow footings on sand.” Geomechanics and Geoengineering, 6(2-2), 91-107.

Uzielli, M., Cassidy, M.J., and Hossain, M.S. 2017. “Bayesian prediction of punch-through for spudcans in stiff-over-soft clay.” ASCE Geotechnical Special Publication GSP286 “Geotechnical Safety and Reliability”, C. Hsein Juang, R.B. Gilbert, L.M. Zhang, J. Zhang, L. Zhang (Eds.), 247-265.

Uzielli, M., Lacasse, S., Nadim, F., and Phoon, K.K. 2007. “*Soil variability analysis for geotechnical practice*.” Keynote paper, In T.S. Tan, K.K. Phoon, D.W. Hight & S. Leroueil (eds.), Proceedings of the 2nd International Workshop on Characterisation and Engineering Properties of Natural Soils. Singapore, November 29 – December 1, 2006. The Netherlands: Taylor & Francis (CD-ROM).

Uzielli, M., Mayne, P.W., and Cassidy, M.J. 2012. “Probabilistic assignment of design strength for sands from in-situ testing data.” Modern Geotechnical Codes of Practice – Development and Calibration. Special Geotechnical Publication, G.A. Fenton, P. Arnold, M.A. Hicks, T. Schweckendiek (Eds.), 214-227, Taylor & Francis.

Vanmarcke, E.H. 1977a. “*Probabilistic modeling of soil profiles*.” Journal of the Geotechnical Engineering Division, 103(11), 1227-1246.

Vanmarcke, E.H. 1977b. “*Reliability of earth slopes*.” Journal of the Geotechnical Engineering Division, 103(11), 1247-1265.

Vanmarcke, E.H. 1983. *Random Fields – Analysis and Synthesis*. MIT Press, Cambridge.

Vick, S. G. 2002. Degrees of belief: subjective probability and engineering judgment. ASCE Press, Reston, Va.

Vick, S.G. 1997. “*Session report - risk analysis practice in different countries*”, In: Risk-Based Dam Safety Evaluation. Proc. Int. Workshop, Trondheim. Norway. Sept. 1997.

Vick, S.G. and Bromwell, L.G. 1989. “*Risk analysis for dam design in karst*.” Journal of Geotechnical Engineering, 115(6), 819-835.

Vick, S.G. and Stewart, R.A. 1996. “*Risk analysis in dam safety practice.*” ASCE Geotechnical Special Publication No. 58, Uncertainty in the Geologic Environment; Prom Theory to Practice. Proc. of Uncertainties '96, Vol. I, 586-603.

Wang, B., Vardon, P.J., and Hicks, M.A. 2016a. “*Investigation of retrogressive and progressive slope failure mechanisms using the material point*.” Computers and Geotechnics, 78, 88-98

Wang, H., Wang, X., Wellmann, J.F., and Liang, R.Y. 2019. “*A Bayesian unsupervised learning approach for identifying soil stratification using cone penetration data*.” Canadian Geotechnical Journal, 56(8), 1184-1205.

Wang, H.J., Zhang, L.M., Luo, H.Y., He, J., and Cheung, R.W.M. 2021a. "*AI-powered landslide susceptibility assessment in Hong Kong*." Engineering Geology, 288(3), 106103.

Wang, H.J., Zhang, L.M., Yin, K.S., Luo, H.Y., and Li, J.H. 2021b. "*Landslide identification using machine learning.*" Geoscience Frontiers, 12, 351-364.

Wang, Y. and Zhao, T. 2017. "*Statistical interpretation of soil property profiles from sparse data using Bayesian compressive sampling*." Géotechnique, 67(6), 523–536.

Wang, Y. B. and E. M. Rathje. 2018. *"Application of a probabilistic assessment of the permanent seismic displacement of a slope*." Journal of Geotechnical and Geoenvironmental Engineering, 144(6), 04018034.

Wang, Y., Au, S.K., and Cao, Z. 2010. "*Bayesian approach for probabilistic characterization of sand friction angles*." Engineering Geology, 114(3), 354-363.

Wang, Y., Cao, Z., and Au, S.K. 2011. *"Practical reliability analysis of slope stability by advanced Monte Carlo simulations in a spreadsheet."* Canadian Geotechnical Journal, 48(1), 162-172.

Wang, Y., Cao, Z., and Li, D. 2016c. "*Bayesian perspective on geotechnical variability and site characterization.*" Engineering Geology, 203, 117-125.

Wang, Y., Hu, Y., and Phoon, K.K. 2021c. "*Non-parametric modelling and simulation of spatiotemporally varying geo-data*." Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, under review.

Wang, Y., Schweckendiek, T., Gong, W.P., Zhao, T.Y., and Phoon, K.K. 2016b. "*Direct probability-based design methods.*" Chapter 7, Reliability of Geotechnical Structures in ISO2394, CRC Press/Balkema, 193-226.

Wastlung, G. 1940. "*Sakerhetsproblem ur praktisk-konstruktiv synpunkt*." Proceedings of The Royal Swedish Institute for Engineering Research, No. 156 (Ingeniors Vetenskaps Akademien), Stockholm.

Welker, A.L. and Gilbert, R.B. 2003. "*The calibration of a flow and transport model with a bench-scale PVD remediation system.*" J. Geotech. Geoenviron. Eng., 129(1), 81-90.

Whitman, R.V. 1984. "*Evaluating calculated risk in geotechnical engineering*." Journal of Geotechnical Engineering, 110(2), 145-188.

Wolff, T. F. 1985. Analysis and design of embankment dam slopes: a probabilistic approach. PhD thesis, Purdue University, Lafayette, Indiana.

Wolff, T. F. 1994. "Evaluating the reliability of existing levees." Report for U.S. Army Engineer Waterways Experiment Station, Geotechnical Laboratory, Vicksburg, Mississippi.

Wolff, T. F. 1995. "Introduction to probability and reliability methods for use in geotechnical engineering." ETL 1110-2-547, US Army Corps of Engineers, Washington DC.

Wolff, T. F. 2008. "Reliability of levee systems." Chapter 12, Reliability-Based Design in Geotechnical Engineering: Computations and Applications, Taylor & Francis, 448-496.

Wu, T. H. 1974. "Uncertainty, safety, and decision in soil engineering." Journal Geotechnical Engineering Division, ASCE, 100(3), 329–348.

Wu, T. H., and Kraft, L. M. 1970a. “Safety analysis of slopes.” Journal of the Soil Mechanics and Foundations Division, 96(2), 609–630.

Wu, T. H., and Kraft, L. M. 1970b. “Seismic safety of earth dams.” Journal of the Soil Mechanics and Foundations Division, 96(6), 1987–2006.

Wu, T.H. 2011. “*2008 Peck Lecture: The observational method: case history and models*.” Journal of Geotechnical and Geoenvironmental Engineering, 137(10), 862-873.

Wu, T.H. and Kraft, L.M. 1967. “*The probability of foundation safety*.” Journal of the Soil Mechanics and Foundations Division, 93(5), 213-231.

Wu, T.H., Lee, I.M., Potter, J.C., and Kjekstad, O. 1987. “*Uncertainties in evaluation of strength of marine sand*.” Journal of Geotechnical Engineering, 113(7), 719-738.

Wu, T.H., Tang, W.H., Sangrey, D.A., and Baecher, G.B. 1989. “*Reliability of offshore foundations – state of the art.*” Journal of Geotechnical Engineering, 115(2), 157-178.

Wu, T.H., Zhou, S.Z., and Gale, S.M. 2007. “*Embankment on sludge: predicted and observed performances*.” Can. Geotech. J., 44, 545-563.

Xiao, S., Zhang, J., Ye, J., and Zheng, J. 2021. “*Establishing region-specific N–Vs relationships through hierarchical Bayesian modeling.*” Engineering Geology, 287, 106105.

Xiao, T., Zou, H.F., Yin, K.S., Du, Y., and Zhang, L.M. (2021). “*Machine learning-enhanced soil classification by integrating borehole and CPTU data with noise filtering*.” Bulletin of Engineering Geology and the Environment, https://doi.org/10.1007/s10064-021-02478-x.

Yan, W.M., Yuen, K.V., and Yoon, G.L. 2009. “*Bayesian probabilistic approach for the correlations of compression index for marine clays*.” Journal of Geotechnical and Geoenvironmental Engineering, 135(12), 1932-1940.

Yang, H.Q., Zhang, L, and Li, D.Q. 2018. “*Efficient method for probabilistic estimation of spatially varied hydraulic properties in a soil slope based on field responses: A Bayesian approach*.” Computers and Geotechnics, 102, 262-272

Yang, X., Han, J., and Parson, R.L. 2010. *Development of Recommended Resistance Factors for Drilled Shafts in Weak Rocks Based on O-cell Tests*. Report No. K-Tran: KU-07-4. Topeka, Kansas: Kansas Department of Transportation.

Yu, X., Abu-Farsakh, M., Hu, Y., Fortier, A.R., and Hasan, M.R. 2017. *Calibration of Geotechnical Axial (Tension and Compression) Resistance Factors (ϕ) for California.* Report No. CA18-2578. Sacramento, CA: California Department of Transportation.

Yucemen, M.S. and Tang, W.H. 1975. “*Long term stability of soil slopes: a reliability approach*.” Proc. of the 2nd Int. Conf. of Statistics and Probability in Soil and Structural Engineering (ICASP), Aachen, 215-229.

Zhang J., Zhang L.M., and Tang W.H. 2009. “*Bayesian framework for characterizing geotechnical model uncertainty.*” Journal of Geotechnical and Geoenvironmental Engineering, 135(7), 932-940.

Zhang, D. M., Du, F., Huang, H., Zhang, F., Ayyub, B. M., and Beer, M. 2018. “Resiliency assessment of urban rail transit networks: Shanghai metro as an example.” Safety Science, 106: 230-243.

Zhang, J., Xian, J.T., Wu, C.G., Zheng, W.T., and Zheng, J.G. 2021. "*Performance-based assessment of permanent displacement of soil slopes using two-dimensional dynamic analysis*." Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, DOI: 10.1080/17499518.2021.2010765

Zhang, J., Boothroyd, P., Calvello, M., Eddleston, M., Grimal, A.C., Iason, P. Luo, Z., Najjar, S., Rodriguez-Marek, A., Straub, D., Uzielli, M., Wang, Y., and Walter, H. 2017. "*Bayesian Method: A Natural Tool for Processing Geotechnical Information*." Joint TC205/TC304 Working Group Report on "Discussion of statistical/reliability methods for Eurocodes". London: International Society of Soil Mechanics and Geotechnical Engineering.

Zhang, J., Juang, C.H., Martin, J.R., and Huang, H.W.2016. "*Inter-region variability of Robertson and Wride method for liquefaction hazard analysis*." Engineering Geology, 203,191-203.

Zhang, J., Li, J.P., Zhang, L.M., and Huang, H.W. 2014. *"Calibrating cross-site variability for reliability-based design of pile foundations."* Computers and Geotechnics, 62, 154-163.

Zhang, J., Tang, W.H., and Zhang, L.M. 2010a. "*Efficient probabilistic back-analysis of slope stability model parameters*." Journal of Geotechnical and Geoenvironmental Engineering, 136(1), 99-109.

Zhang, J., Zhang, L.M., and Tang, W.H. 2011. "*New methods for system reliability analysis of soil slopes.*" Canadian Geotechnical Journal, 48(7), 1138-1148.

Zhang, L., Li, J., Li, X., Zhang, J., and Zhu, H. 2016. *Rainfall-Induced Soil Slope Failure: Stability Analysis and Probabilistic Assessment*. London: CRC Press.

Zhang, L.L., Zhang, J., and Zhang, L.M. 2012. *Selected Works of Professor Wilson Tang and Presentations of Professor Wilson Tang Memorial Symposium*. In CD Rom, Hong Kong University of Science and Technology.

Zhang, L.L., Zhang, J., Zhang, L.M., and Tang, W.H. 2010b. "*Back analysis of slope failure with Markov chain Monte Carlo simulation.*" Computers and Geotechnics, 37(7-8), 905-912.

Zhang, L.M. 2004. "*Reliability verification using proof pile load tests*." J. Geotech. Geoenviron. Eng., 130(11), 1203-1213.

Zhang, L.M. and Chu, L.F. 2009a. "*Calibration of methods for designing large-diameter bored piles: Ultimate limit state.*" Soils and Foundations, 49(6), 883-896.

Zhang, L.M. and Chu, L.F. 2009b. "*Calibration of methods for designing large-diameter bored piles: Serviceability limit state.*" Soils and Foundations, 49(6), 897-908.

Zhang, W., Li, H., Li, Y., Liu, H., Chen, Y., and Ding, X. (2021). Application of deep learning algorithms in geotechnical engineering: a short critical review. Artificial Intelligence Review, 54, 5633–5673.

Zhang, Y. and Huang, H.W. 2010. Code for Risk Management of Underground Works in Urban Rail Transit (in Chinese) (GB50652-2011). Ministry of Housing and Urban-Rural Development of the People's Republic of China.

Zheng, D., Huang, J., Li, D.Q., Kelly, R., and Sloan, S. 2018. "Embankment prediction using testing data and monitored behaviour: A Bayesian updating approach." *Computers and Geotechnics* 93: 150-162.